Given : - tree T (2-state p.p)

- matrix $A = [a_{pi}]$ alt reads

$R = [r_{pi}]$ ref reads

$D = [d_{pi}]$ s.t. $d_{pi} = a_{pi} + r_{pi}$

$A \sim \text{Binom}(D, F)$

$$\Pr(A \mid D, F) = \prod_{p=1}^{m} \prod_{i=1}^{n} \Pr(a_{pi} \mid d_{pi}, f_{pi})$$

$$= \prod_{p=1}^{m} \prod_{i=1}^{n} \binom{d_{pi}}{a_{pi}} \cdot f_{pi}^{a_{pi}} \cdot (1 - f_{pi})^{r_{pi}}$$

Given A & D, $\hat{F}$ that maximizes $\Pr(A \mid D, F)$

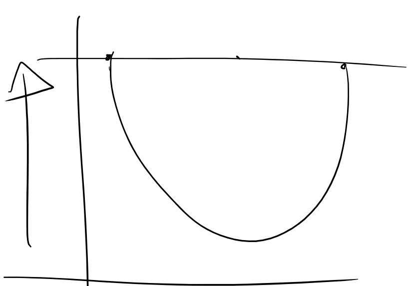
$$\hat{f}_{pi} = \frac{a_{pi}}{d_{pi}}$$

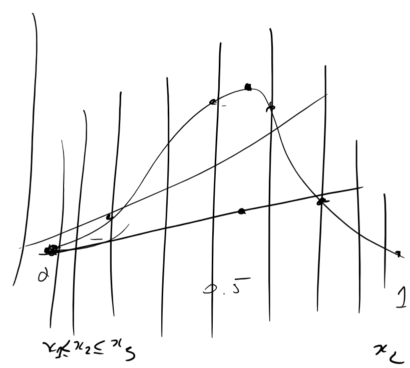
$$\max_{F} \sum_{p} \sum_{i} \left[\cancel{\log \binom{d_{pi}}{a_{pi}}} + \underbrace{a_{pi} \log(f_{pi}) + r_{pi} \log(1 - f_{pi})}_{g(a, r, f)} \right]$$

s.t. $f_{pi} \geq \sum_{j \in \delta(i)} f_{pj}$ $\forall p \in [m]$

$\varepsilon > 0$

$$g(a, r, \boxed{f}) = a \log(f) + r \log(1 - f) \qquad f \in [\varepsilon, 1 - \varepsilon]$$





$a = 50$
$r = 50$

$g(a, r, x_1)$ $g(a, r, x_L)$

$x_1, \ldots, x_L$

$x_1 = \varepsilon$, $x_L = 1 - \varepsilon$

λ-method

λ_1, λ_2 λ_L

$x_1 = \varepsilon$
$x_2 = 0.1$
$x_3 = 0.2$
$f = 0.15$
$\lambda_2 = 0.5$
$\lambda_3 = 0.5$
$\vdots$
$x_{L-1} = 0.5$
$x_L = 1 - \varepsilon$

$$\forall i, p \quad \begin{cases} \lambda_\ell \geq 0 & \forall \ell \in \{1, \ldots, L\} \\ \sum_{\ell=1}^{L} \lambda_\ell = 1 \\ \sum_{\ell=1}^{L} \lambda_\ell \cdot x_\ell = f \end{cases}$$

$$\max \sum_{p=1}^{m} \sum_{i=1}^{n} \sum_{\ell=1}^{L} g(a_{pi}, r_{pi}, x_\ell) \cdot \boxed{\lambda_{p,i,\ell}}$$

s.t. $f_{pi} \geq \sum_{j \in \delta(i)} f_{pj}$ $\forall p \in \{1, \ldots, m\} = [m]$

$\sum_{\ell} \ldots = 1$ $\forall p \in [m]$, $i \in [n]$

$$\text{s.t.} \quad f_{pi} \geq \sum_{j \in \delta(i)} f_{pj} \qquad \forall p \in \{1, \ldots, m\} = [m]$$

$$\sum_{\ell=1}^{L} x_{i,p,\ell} = 1 \qquad \forall p \in [m],\ i \in [n]$$

$$\sum_{\ell=1}^{L} x_{i,p,\ell}\, x_L = f_{pi} \qquad \forall p \in [m],\ i \in [n]$$

$$x_{i,p,\ell} \geq 0 \qquad \forall p \in [m],\ i \in [n],\ \ell \in [L]$$

$$0 \leq f_{pi} \leq 1 \qquad \forall p,\ i$$