

# CS 598MEB

# Computational Cancer Biology

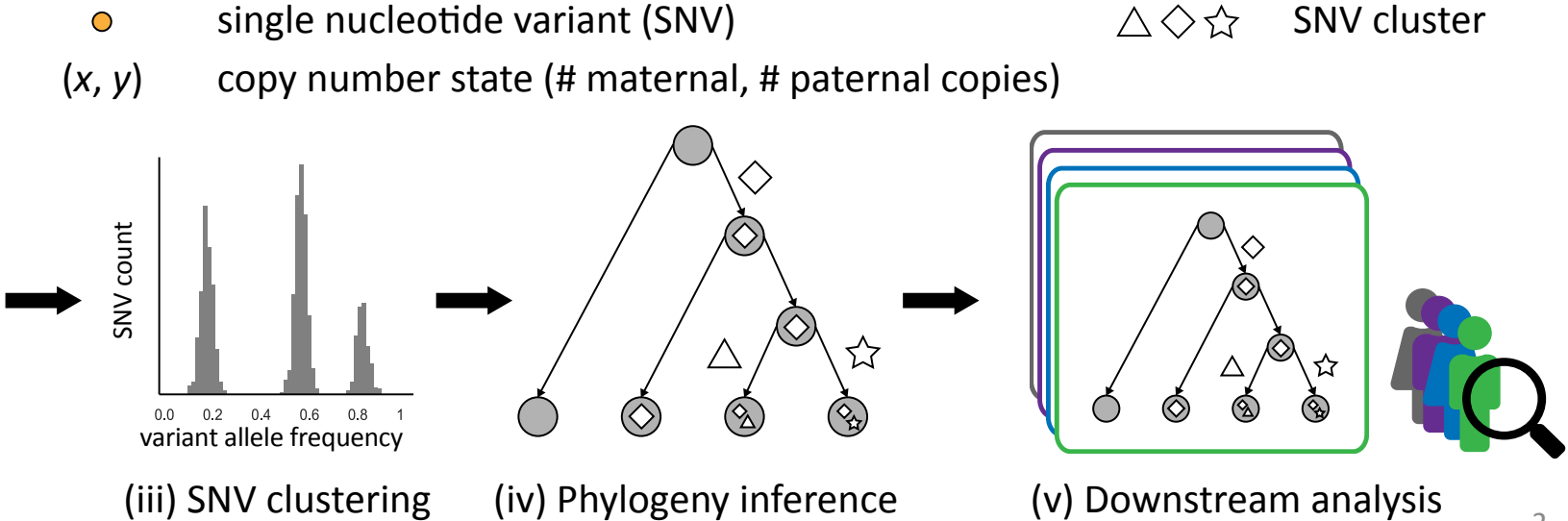
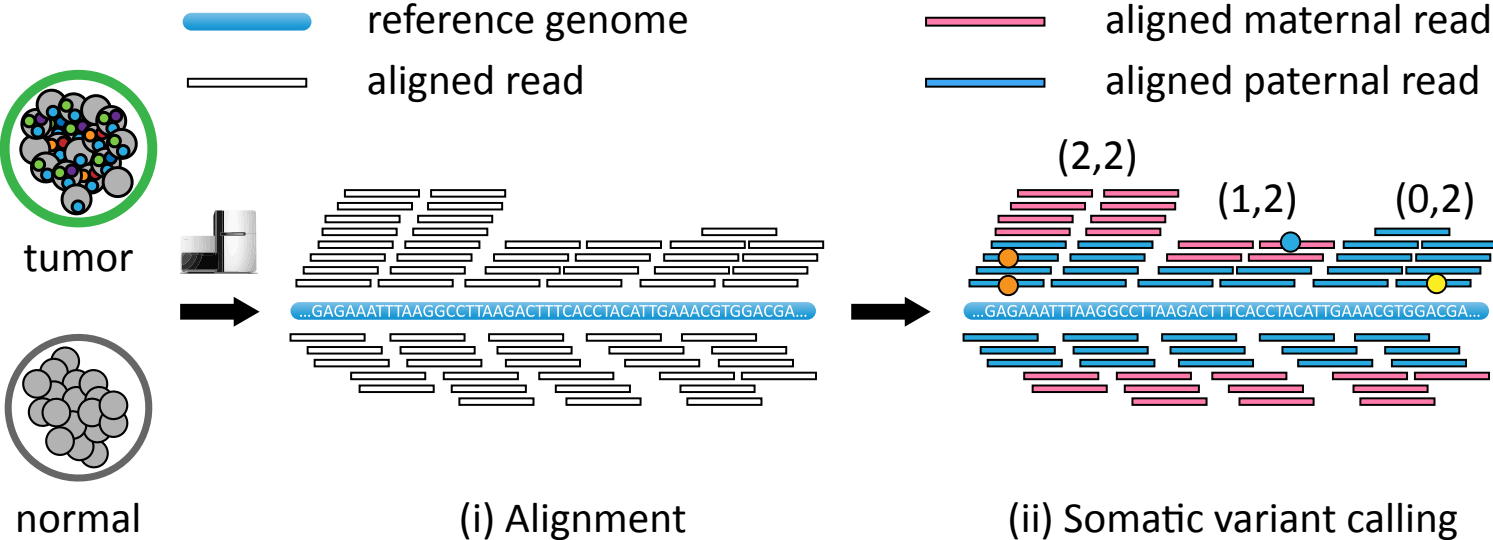
## Lecture 11

Mohammed El-Kebir

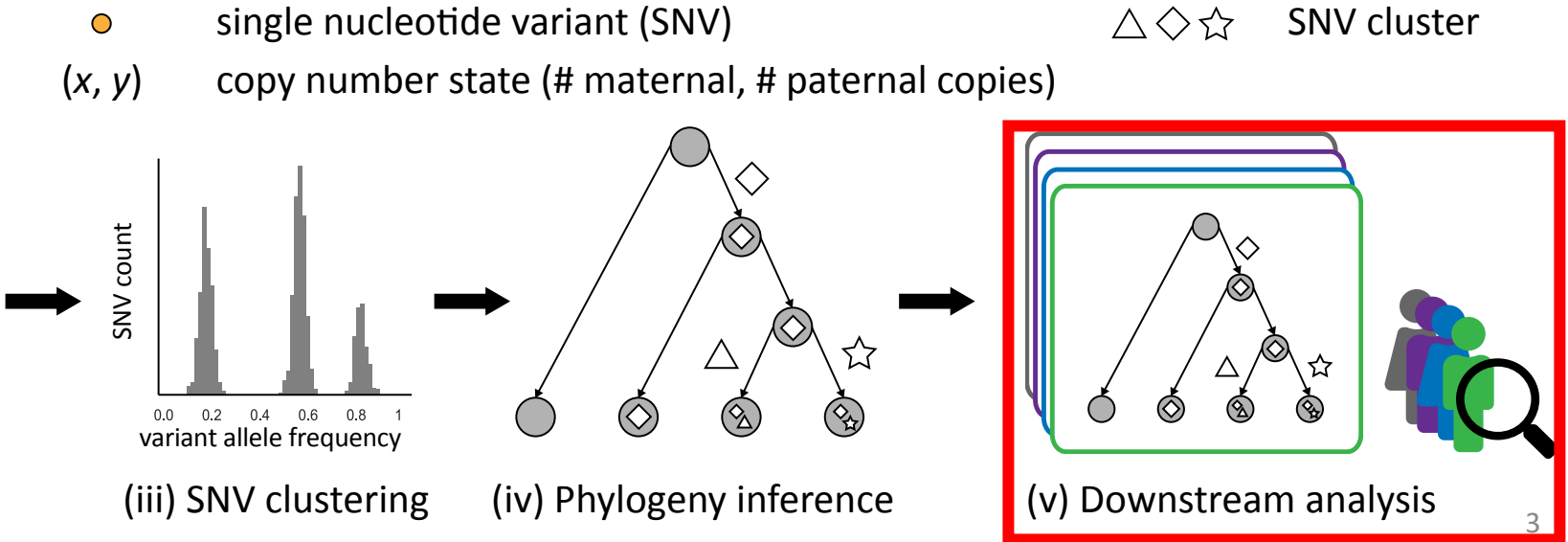
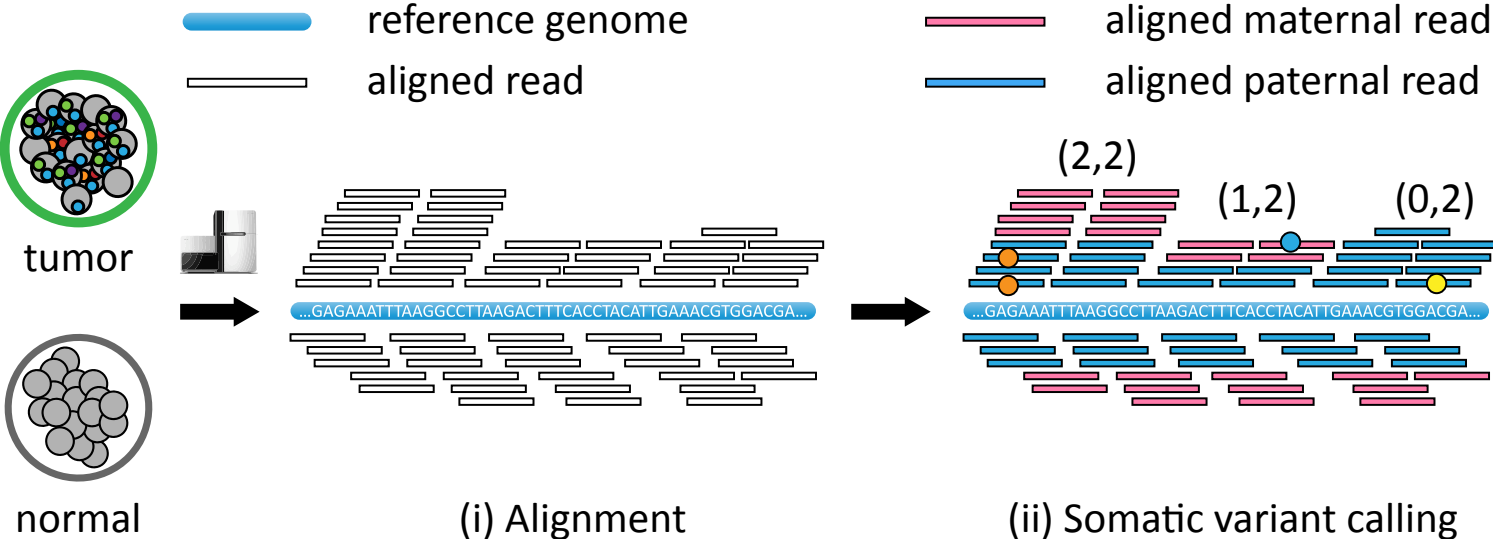
February 25, 2020



# Cancer Phylogenetics Pipeline



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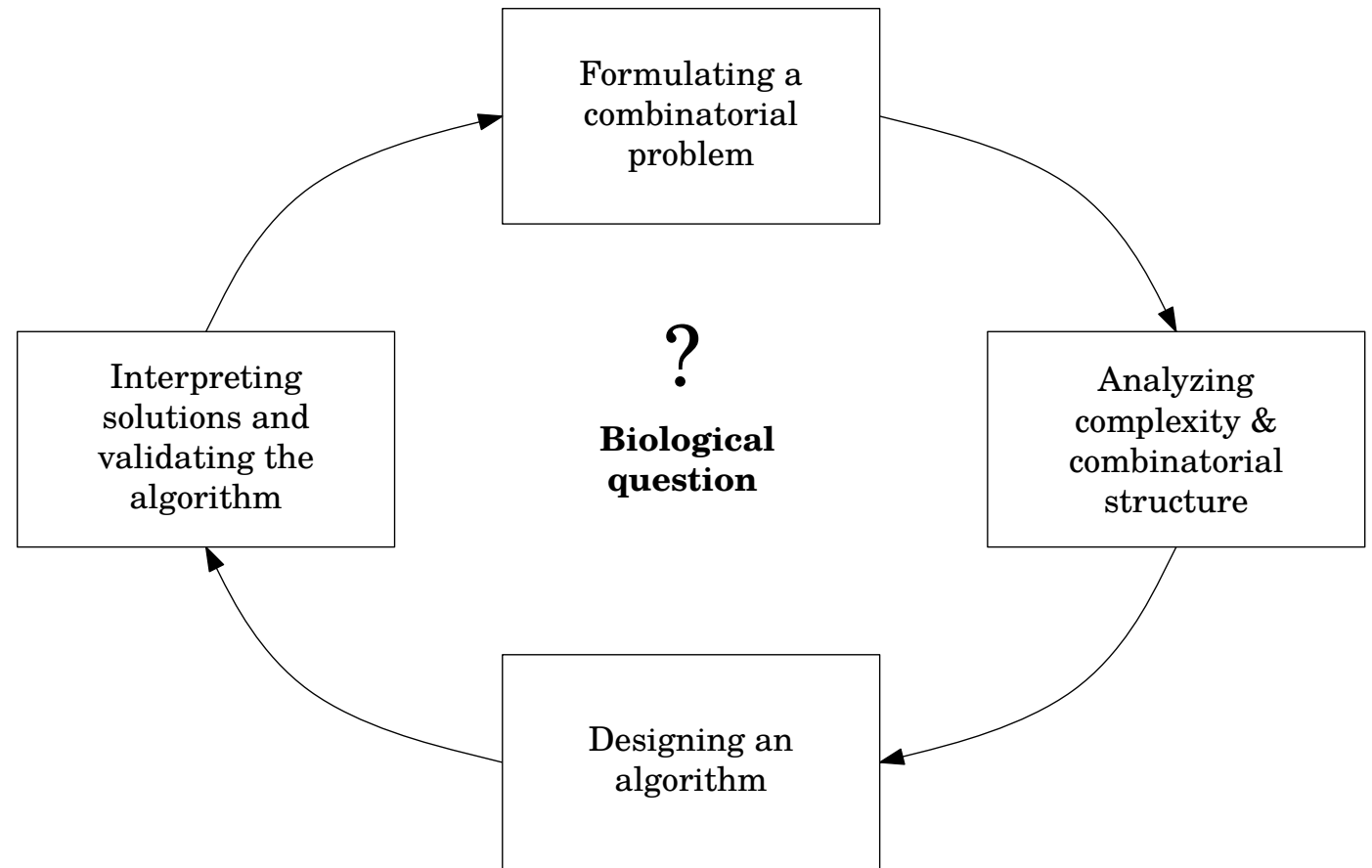


# Outline

- Metastasis
- Maximum parsimony
- Problem statement
- Complexity
- Algorithm & results
- Problem variants

## Reading:

- **M. El-Kebir**, G. Satas and B.J. Raphael. Inferring parsimonious migration histories for metastatic cancers. [Nature Genetics, 50:718-726, 2018.](#)
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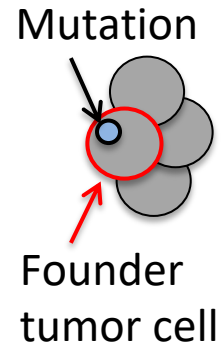




# Tumorigenesis: (i) Cell Mutation

## Clonal Theory of Cancer

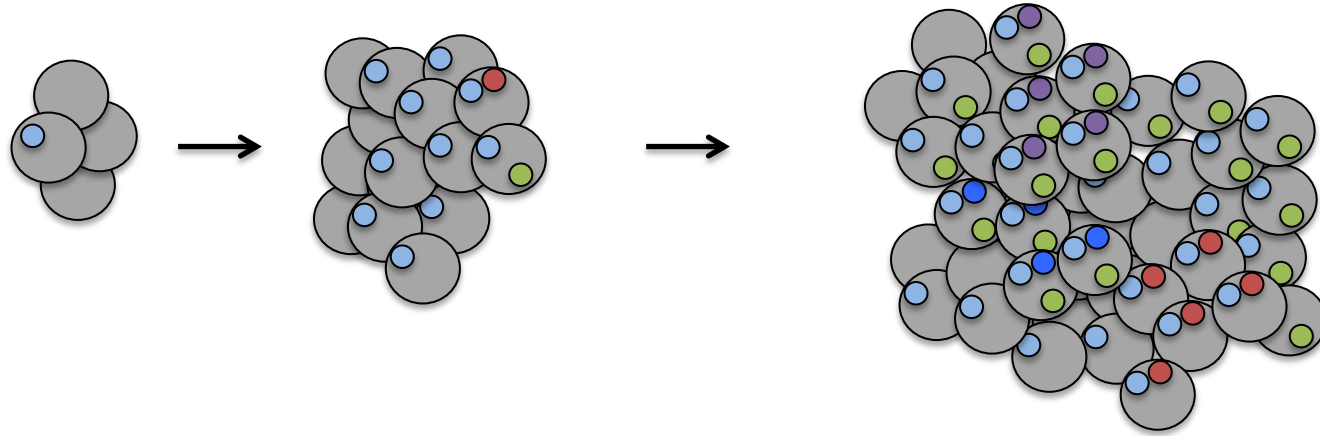
[Nowell, 1976]



# Tumorigenesis: (i) Cell Mutation, (ii) Cell Division

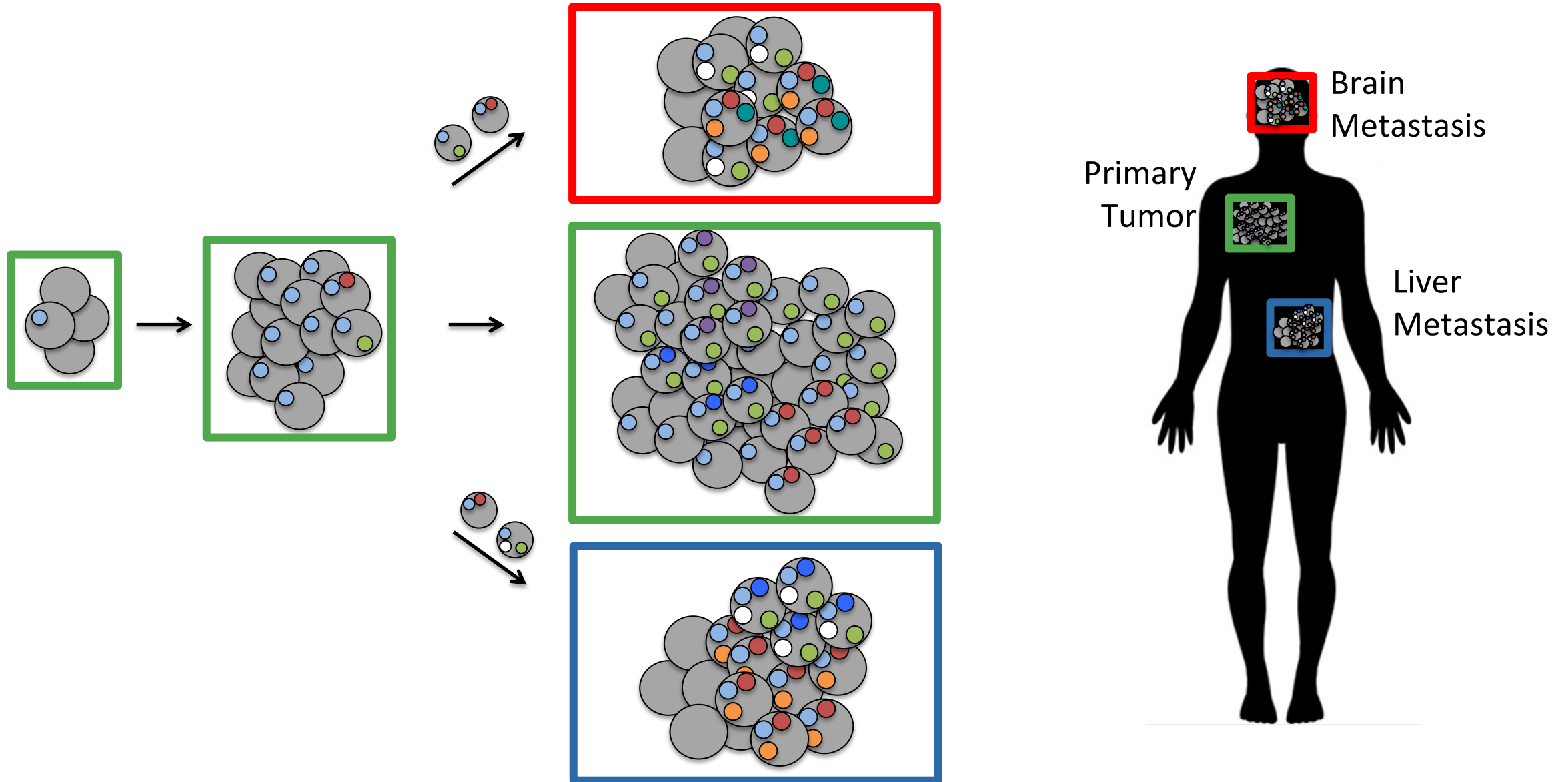
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[Nowell, 1976]

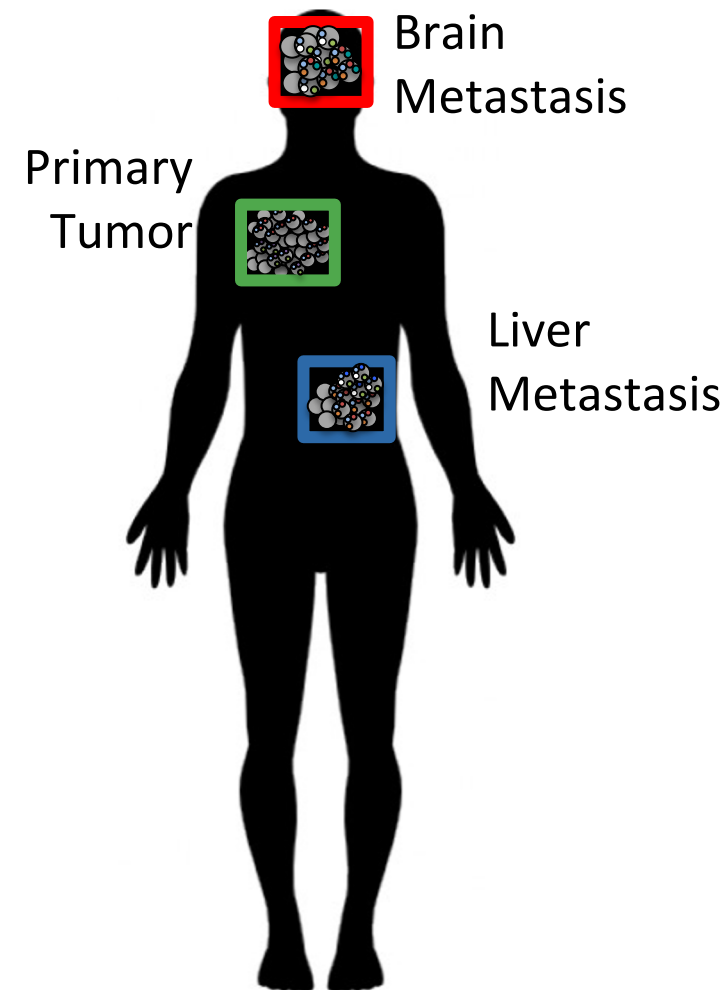
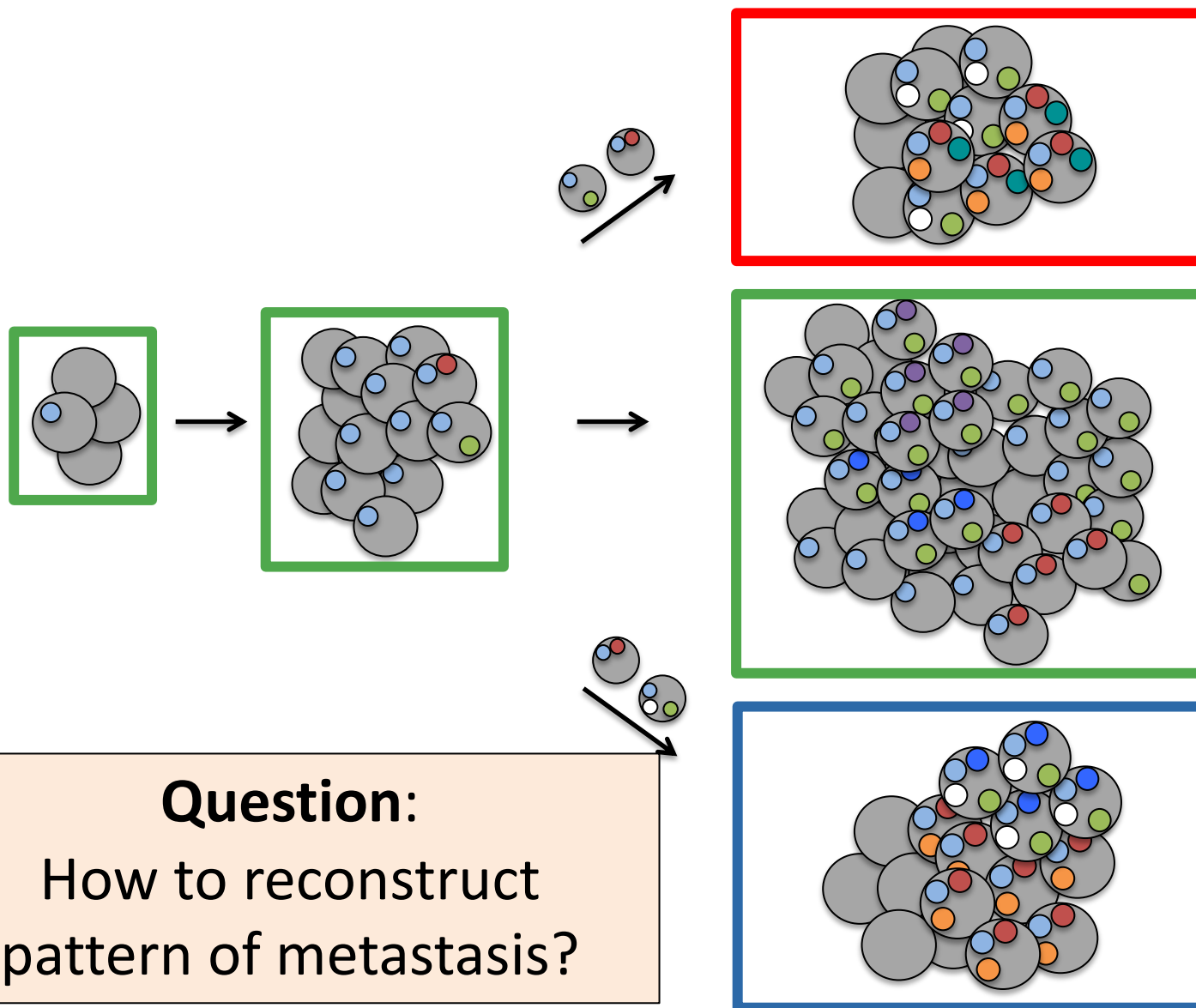


Heterogeneous Tumor

# Tumorigenesis: (i) Cell Division, (ii) Mutation & (iii) Migration

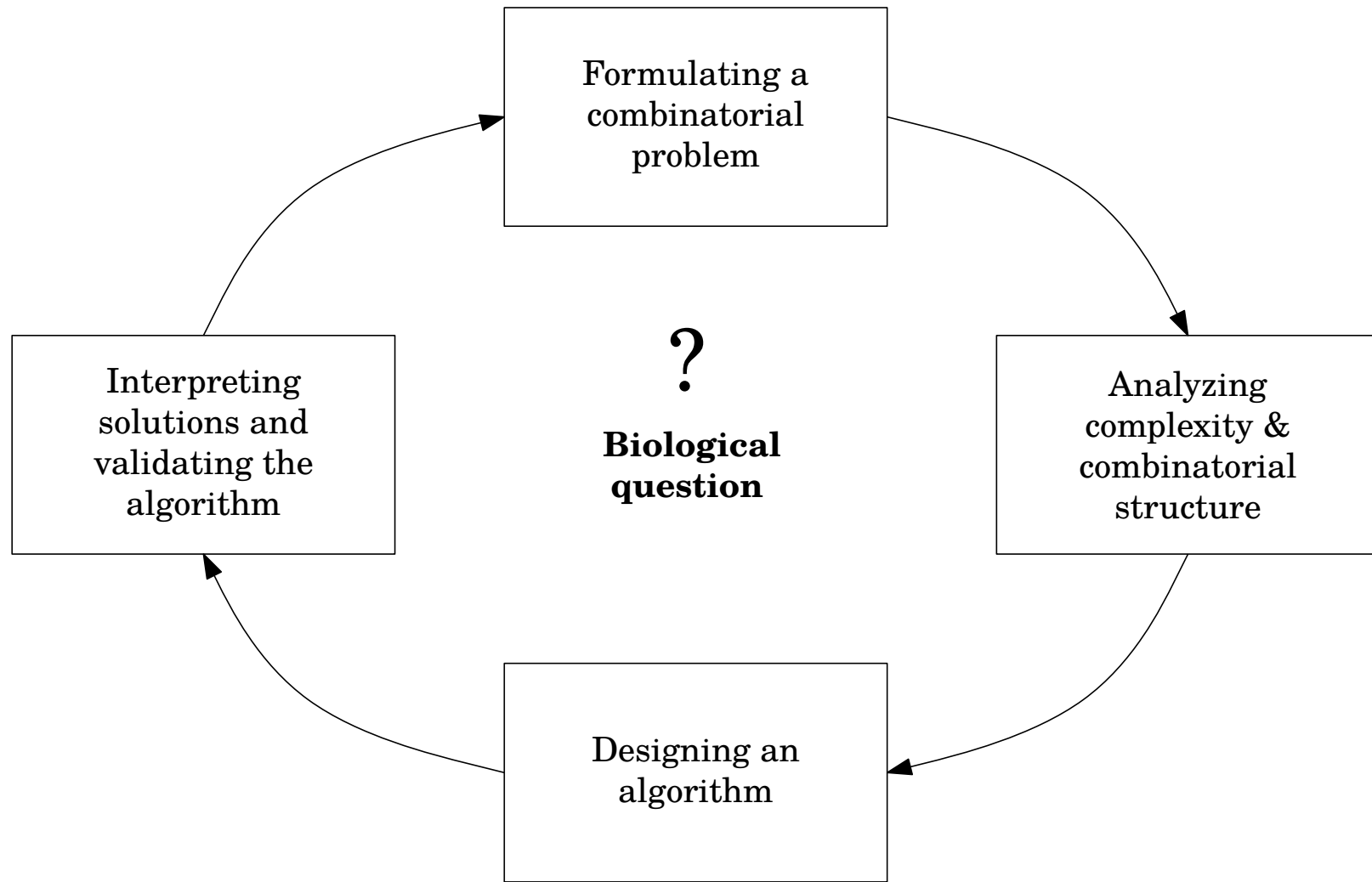


# Tumorigenesis: (i) Cell Division, (ii) Mutation & (iii) Migration



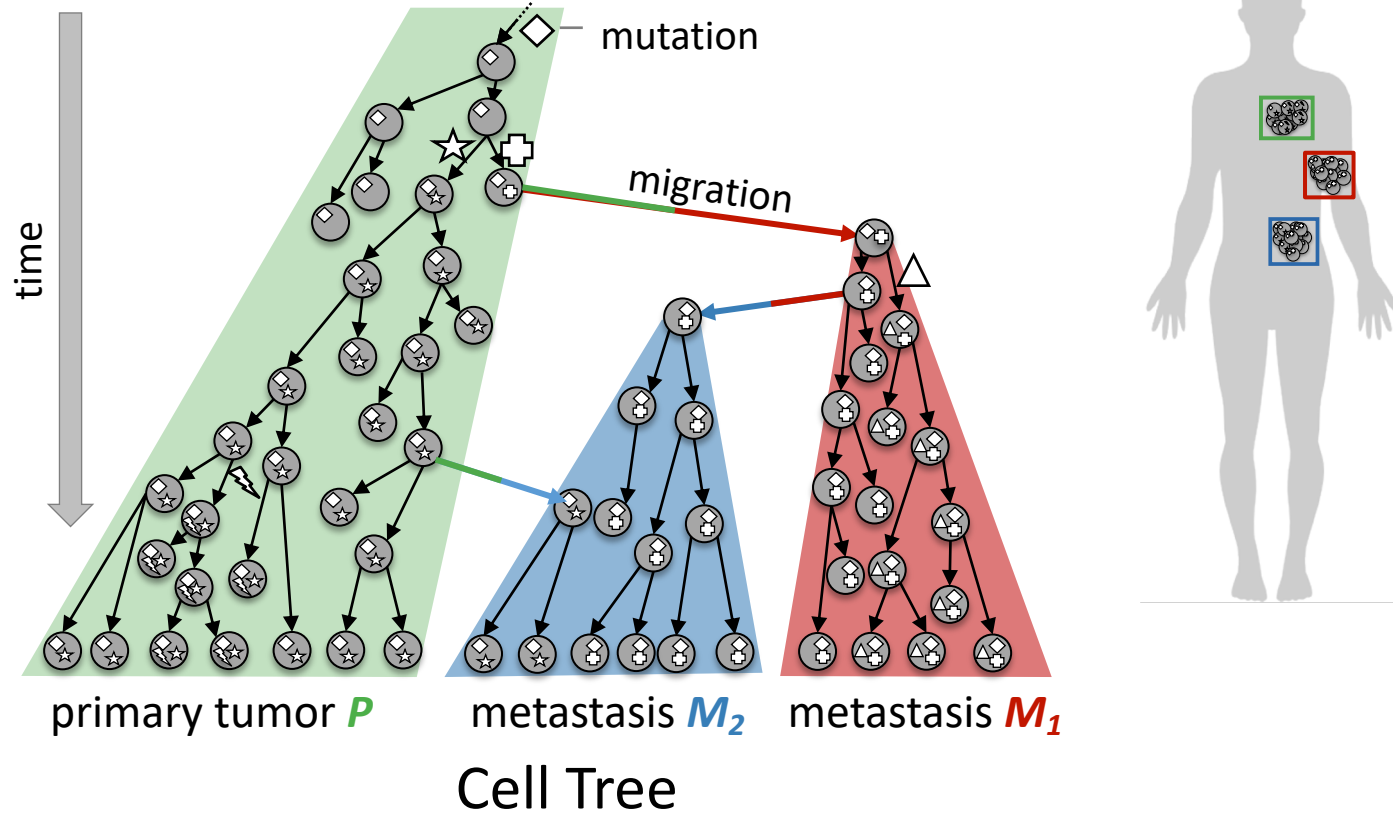
**Question:**  
How to reconstruct  
pattern of metastasis?

# Key Challenge in Computational Biology

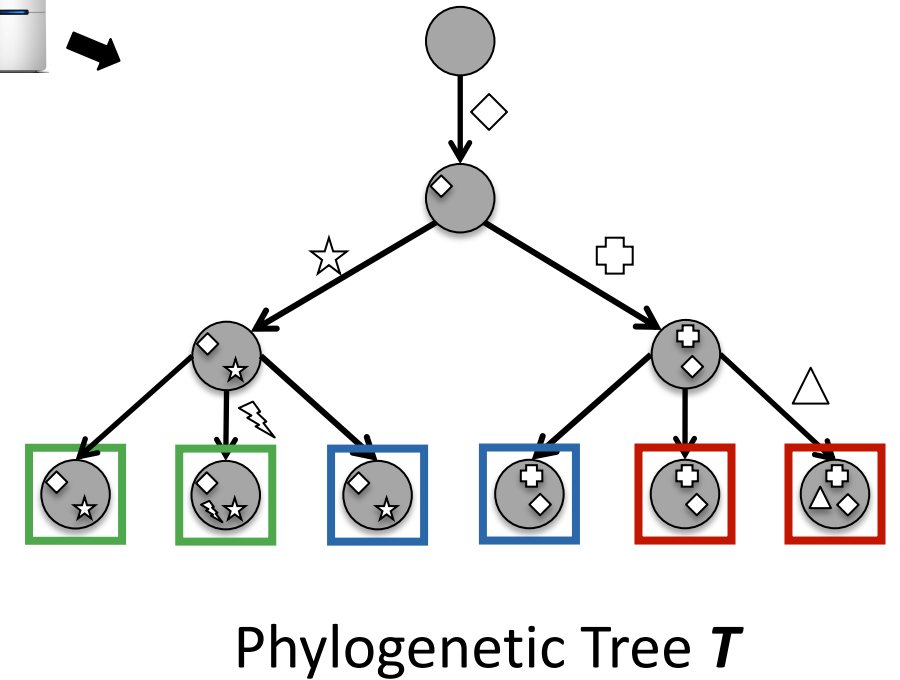
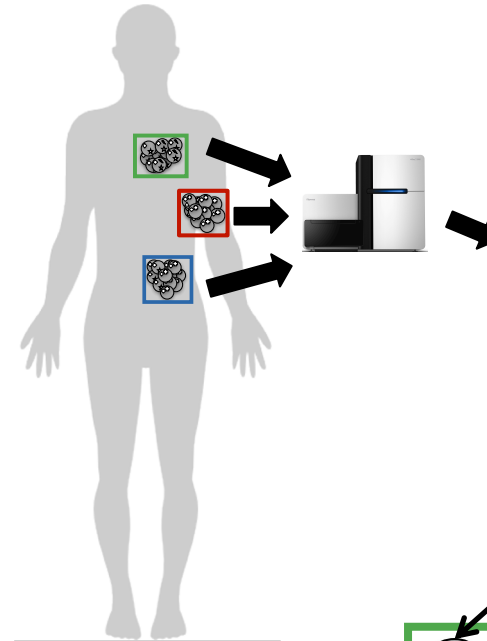
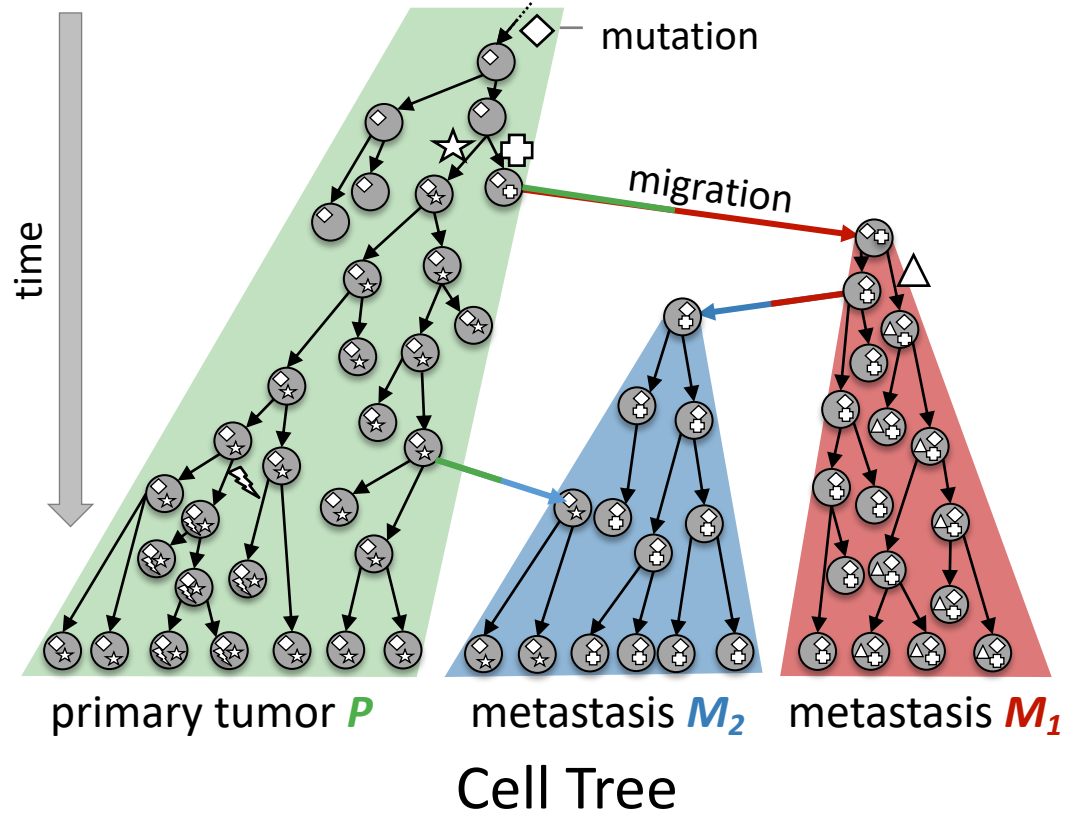


Translating a biological problem into computer science

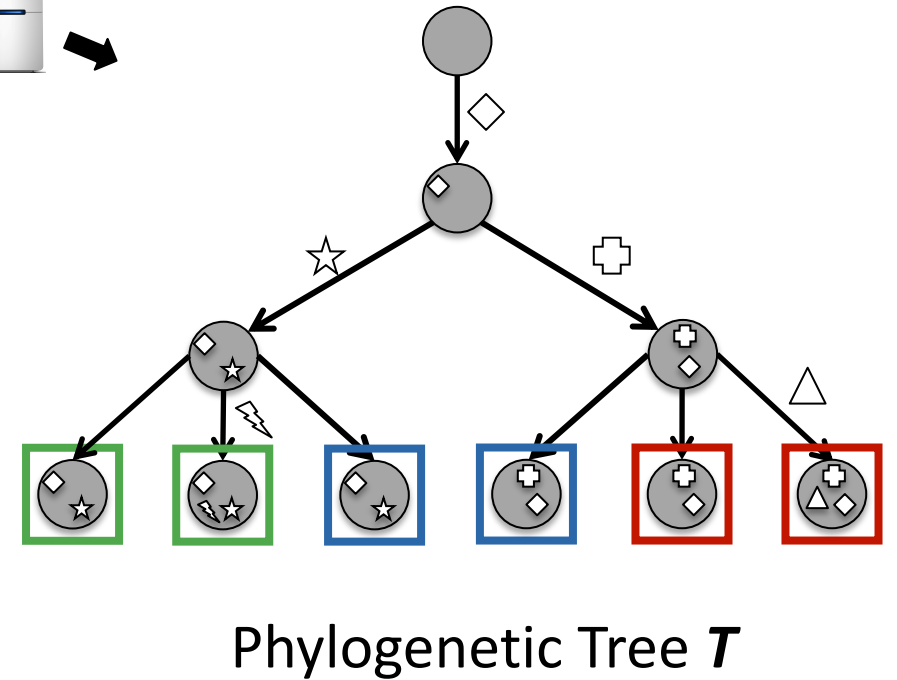
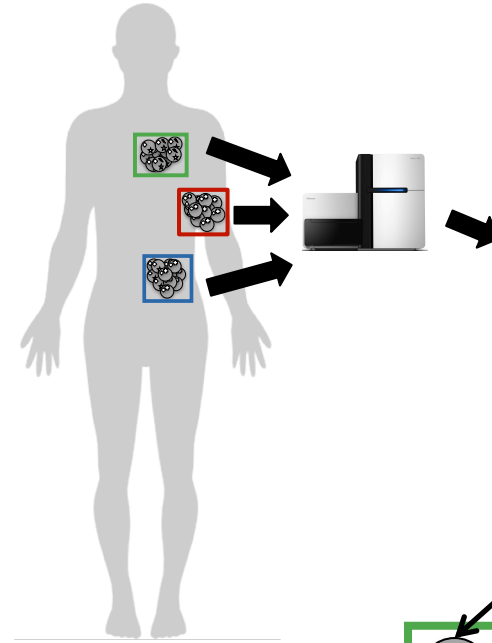
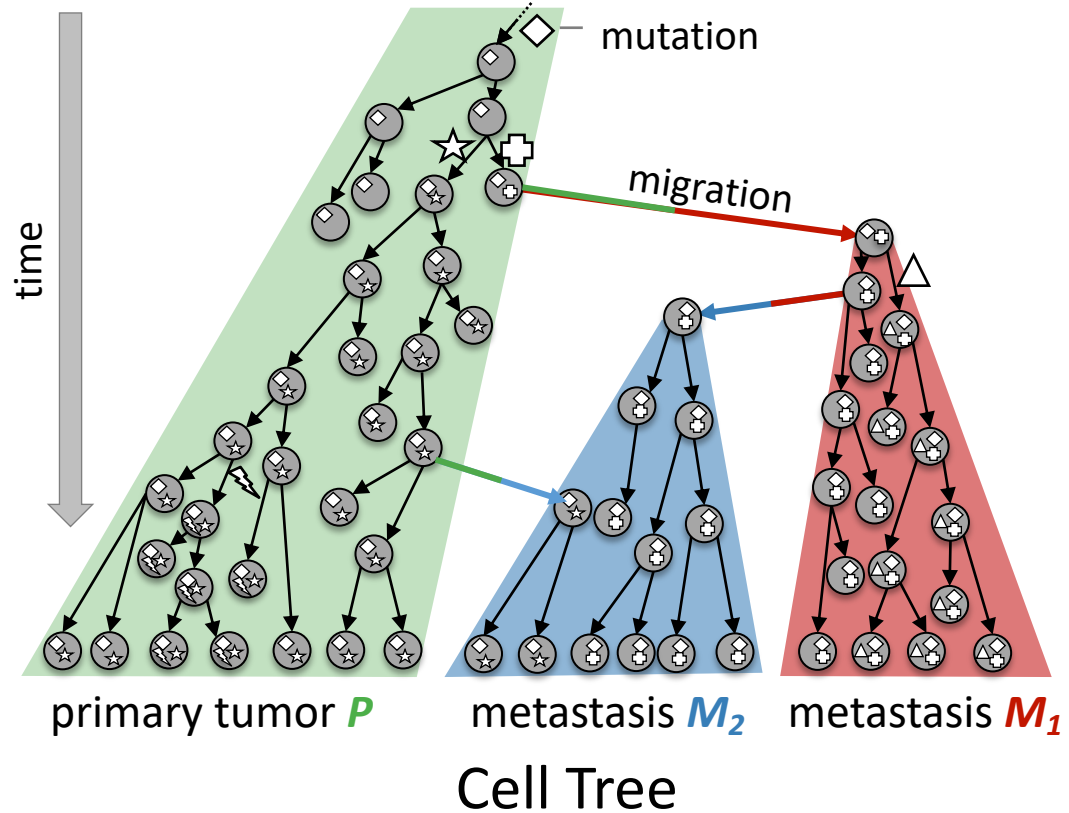
# Tumorigenesis: (i) Cell Division, (ii) Mutation & (iii) Migration



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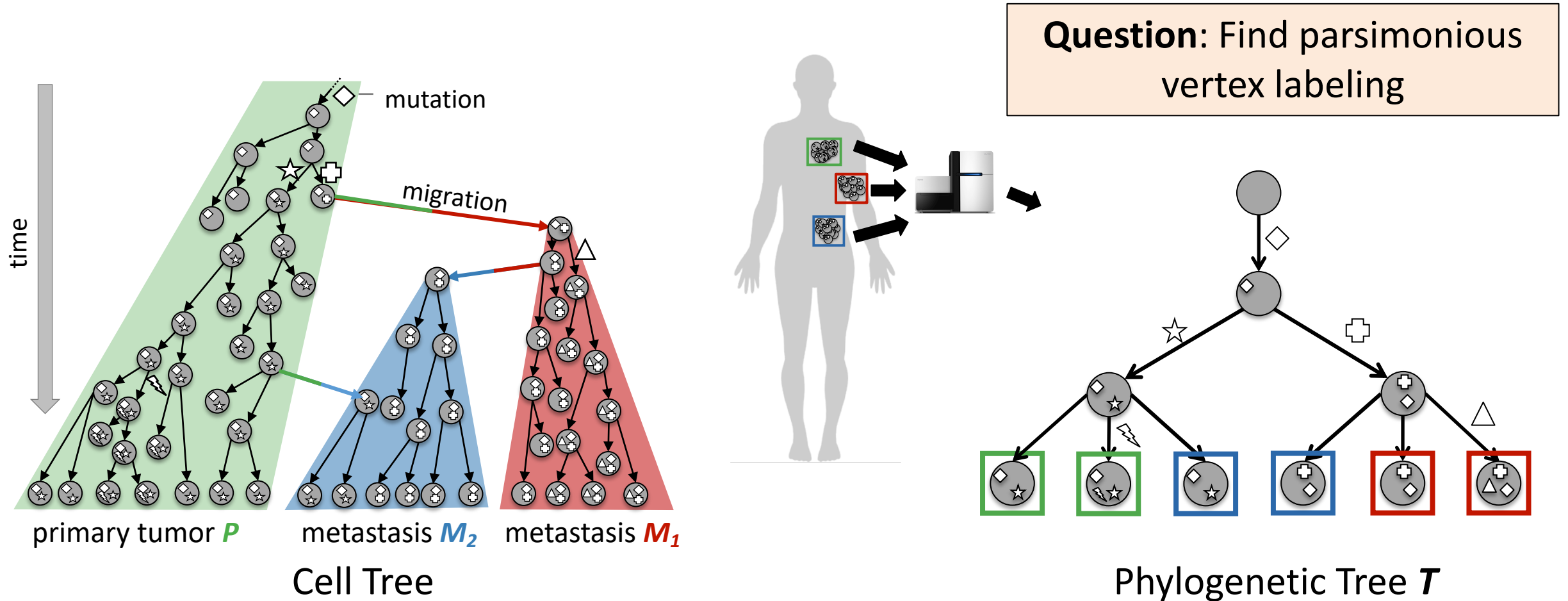
# Tumorigenesis: (i) Cell Division, (ii) Mutation & (iii) Migration



**Goal:** Given phylogenetic tree  $T$ , find *parsimonious* vertex labeling  $\ell$  with fewest migrations

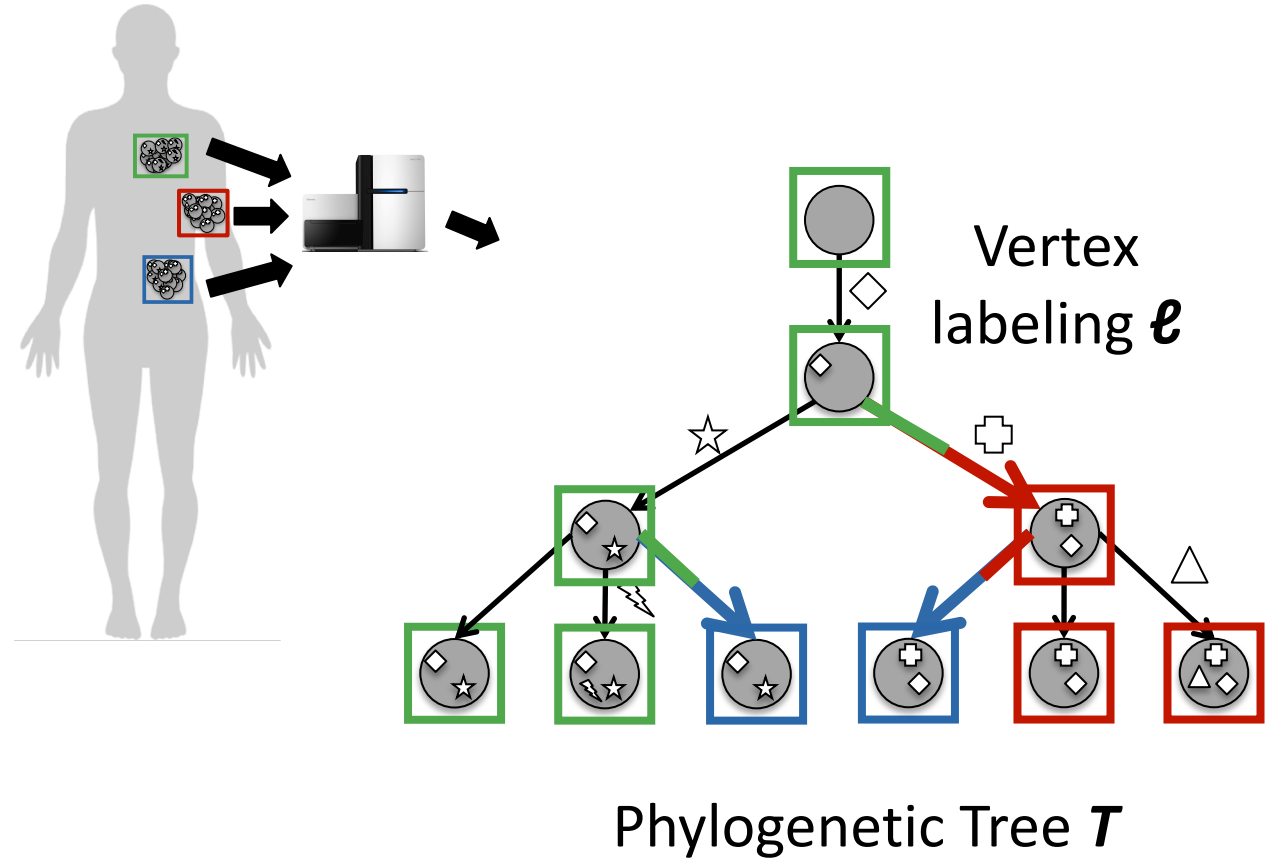
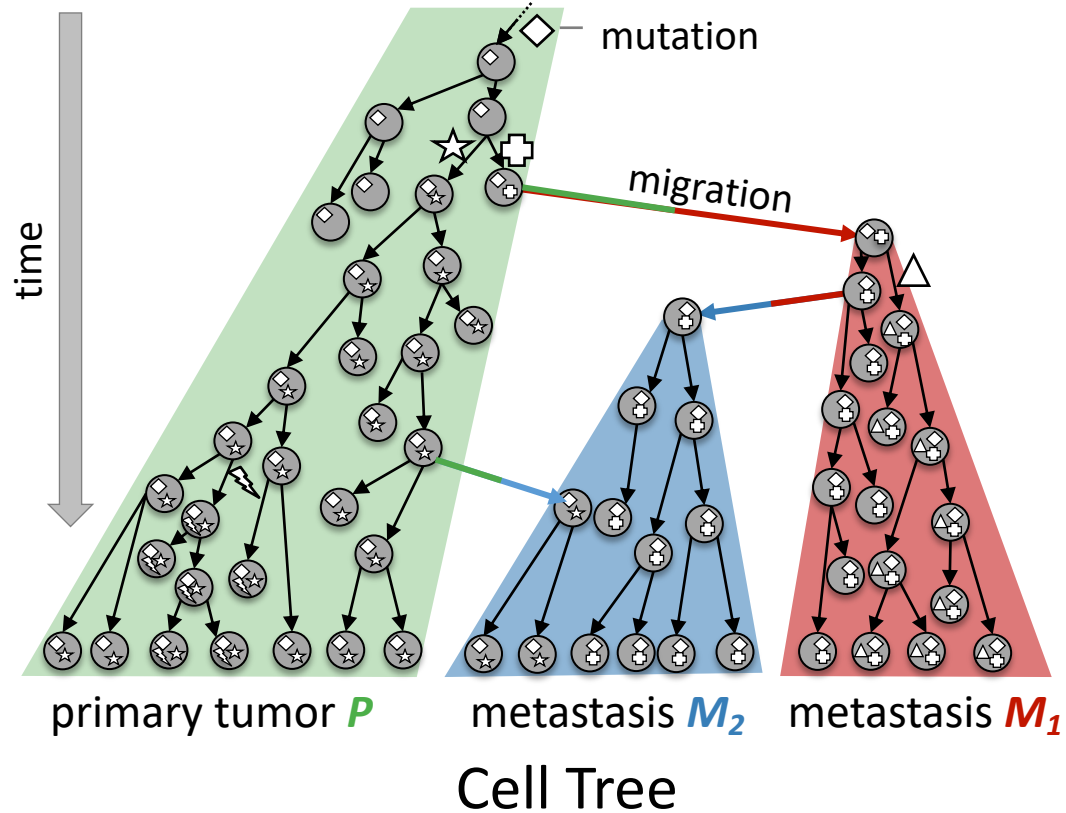


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# Tumorigenesis: (i) Cell Division, (ii) Mutation & (iii) Migration



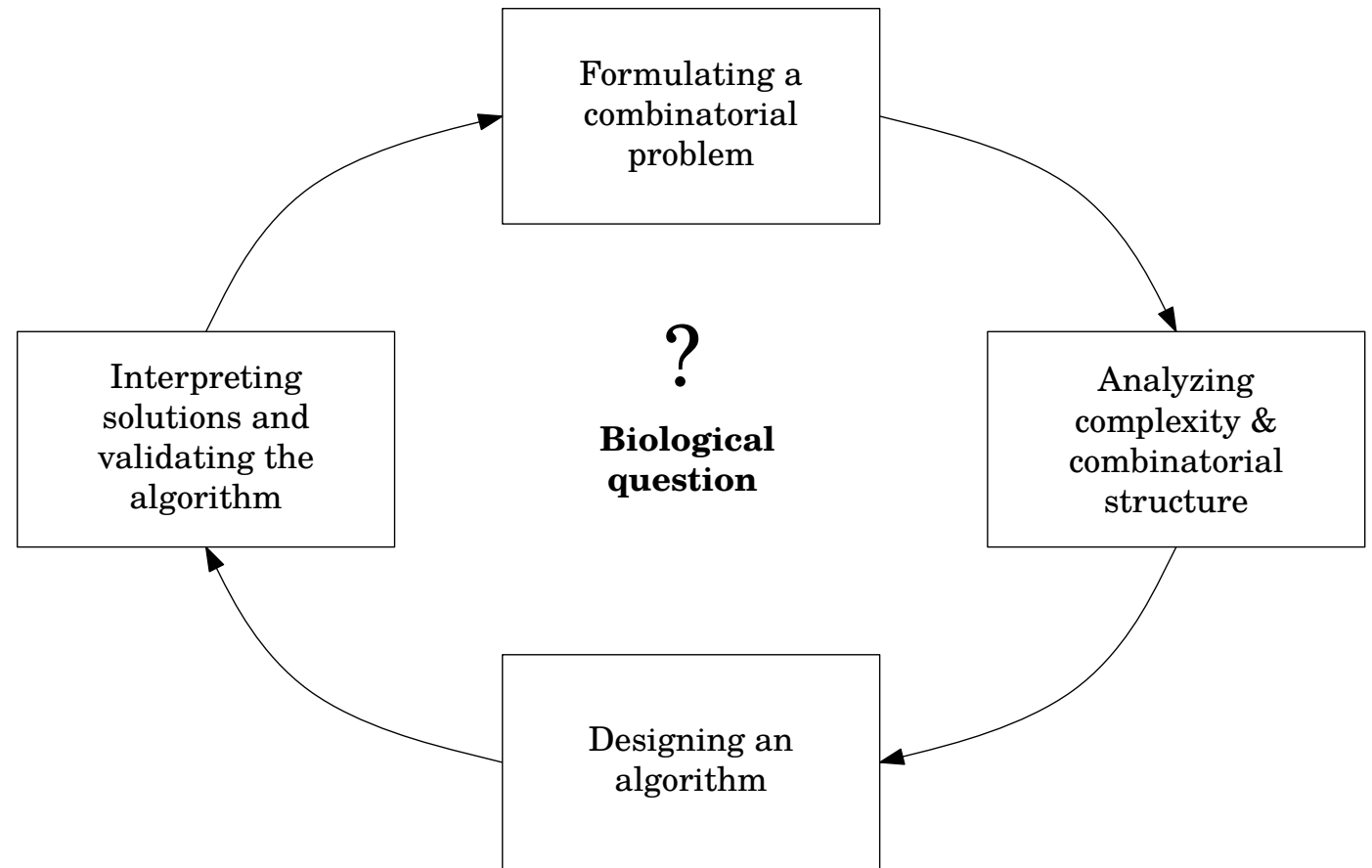
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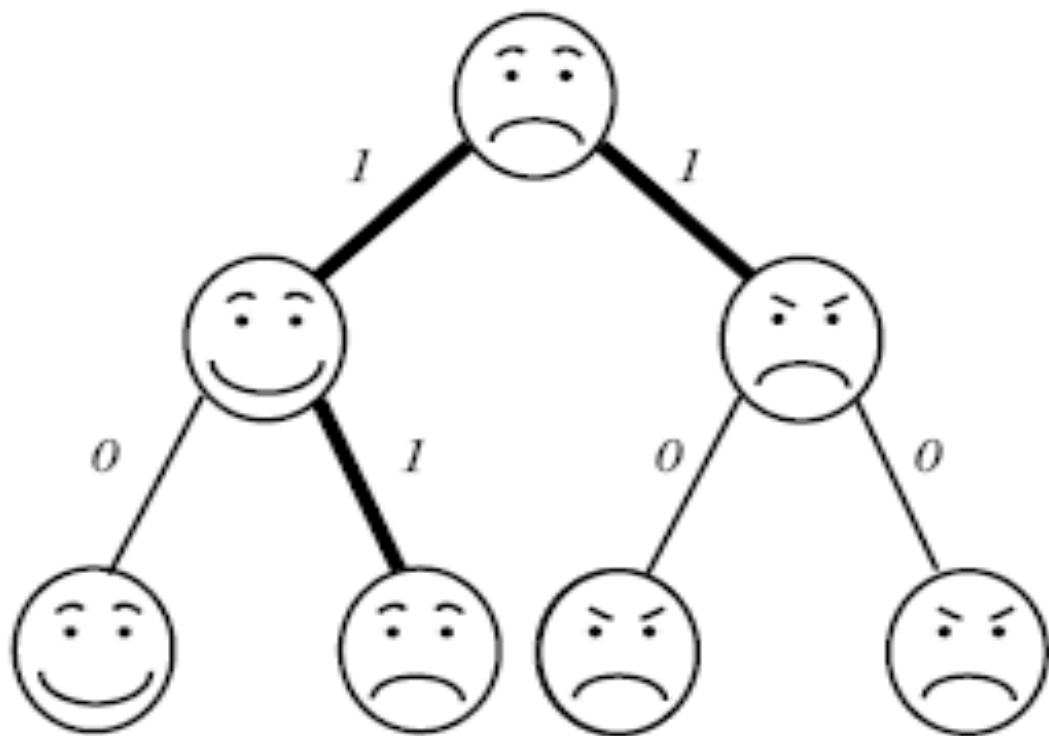
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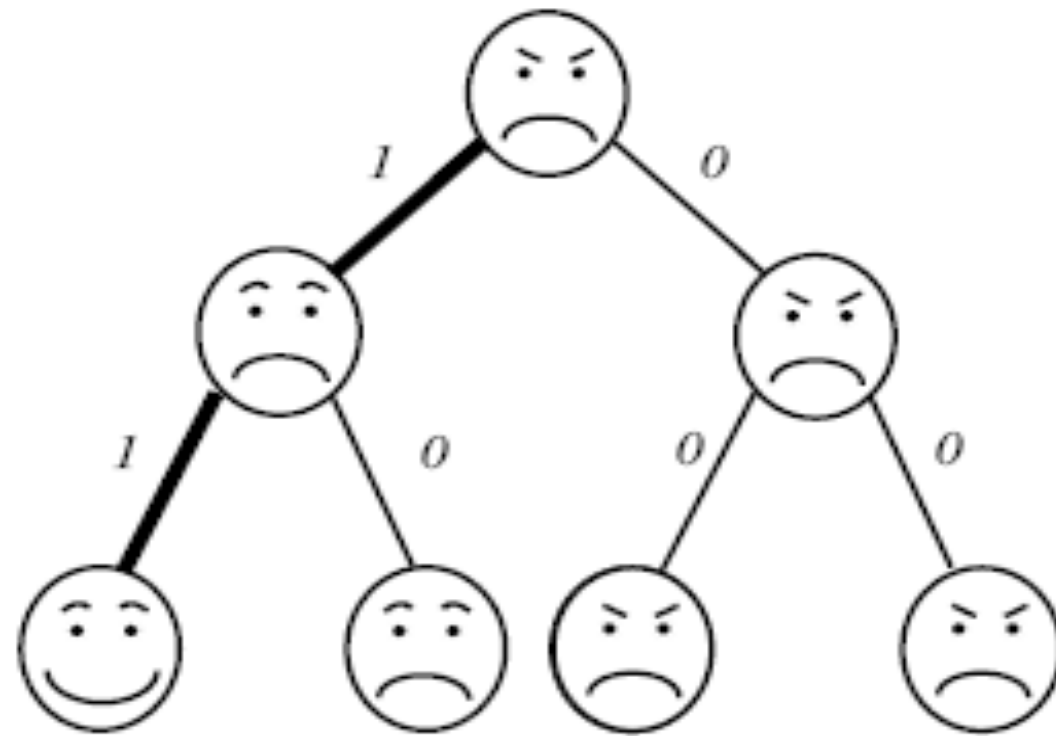
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# Character-Based Phylogeny Reconstruction: Criterion



(a) *Parsimony Score=3*



(b) *Parsimony Score=2*

**Parsimony:** minimize number of changes on edges of tree

# A Small and a Large Problem

## **Small Maximum Parsimony Phylogeny Problem:**

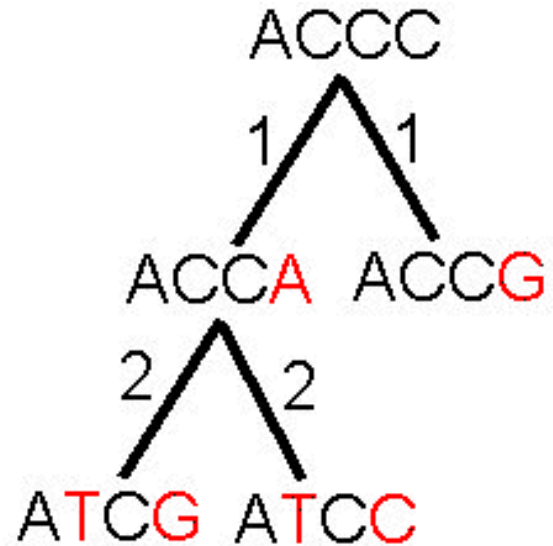
Given  $m \times n$  matrix  $A = [a_{i,j}]$  and tree  $T$  with  $m$  leaves, find assignment of character states to each internal vertex of  $T$  with minimum parsimony score.

## **Large Maximum Parsimony Phylogeny Problem:**

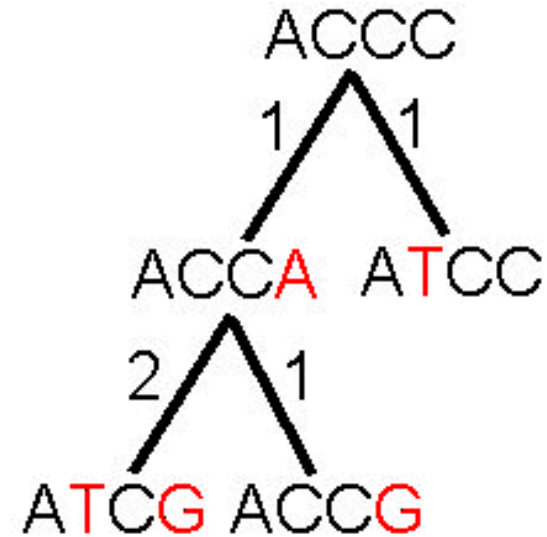
Given  $m \times n$  matrix  $A = [a_{i,j}]$ , find a tree  $T$  with  $m$  leaves labeled according to  $A$  and an assignment of character states to each internal vertex of  $T$  with minimum parsimony score.

**Question:** Are both problems easy (i.e. in P)?

# Small Maximum Parsimony Phylogeny Problem



**Less**  
**Parsimonious**  
**Score: 6**



**More**  
**Parsimonious**  
**Score: 5**

**Key observations:** (1) Characters can be solved independently.  
(2) Optimal substructure in subtrees.

# Recurrence for Small Maximum Parsimony Problem

## **Small Maximum Parsimony Phylogeny Problem:**

Given rooted tree  $T$  whose leaves are labeled by  $\sigma : L(T) \rightarrow \Sigma$ , find assignment of states to each internal vertex of  $T$  with minimum parsimony score.

# Recurrence for Small Maximum Parsimony Problem

## **Small Maximum Parsimony Phylogeny Problem:**

Given rooted tree  $T$  whose leaves are labeled by  $\sigma : L(T) \rightarrow \Sigma$ , find assignment of states to each internal vertex of  $T$  with minimum parsimony score.

Let  $\mu(v, s)$  be the minimum number of mutations in the subtree rooted at  $v$  when assigning state  $s$  to  $v$ .

Let  $\delta(v)$  be the set of children of  $v$ .



# Recurrence for Small Maximum Parsimony Problem

## Small Maximum Parsimony Phylogeny Problem:

Given rooted tree  $T$  whose leaves are labeled by  $\sigma : L(T) \rightarrow \Sigma$ , find assignment of states to each internal vertex of  $T$  with minimum parsimony score.

Let  $\mu(v, s)$  be the minimum number of mutations in the subtree rooted at  $v$  when assigning state  $s$  to  $v$ .

$$c(s, t) = \begin{cases} 0, & \text{if } s = t \\ 1, & \text{if } s \neq t, \end{cases}$$

Let  $\delta(v)$  be the set of children of  $v$ .

$$\mu(v, s) = \min \begin{cases} \infty, & \text{if } v \in L(T) \text{ and } s \neq \sigma(v), \\ 0, & \text{if } v \in L(T) \text{ and } s = \sigma(v), \\ \sum_{w \in \delta(v)} \min_{t \in \Sigma} \{c(s, t) + \mu(w, t)\}, & \text{if } v \notin L(T). \end{cases}$$

# Filling out DP Table and Traceback

Filling out  $\mu$        $\text{Fill}(T, r(T), \sigma, \Sigma)$

$\text{Fill}(T, v, \sigma, \Sigma)$        $O(m|\Sigma|^2)$

if  $v \in \mathcal{L}(T)$  then

For  $s \in \Sigma$

if  $s = \sigma(v)$  then

$$\mu(v, s) = 0$$

else  $\mu(v, s) = \infty$

else

For  $w \in \delta(v)$

$\text{Fill}(T, w, \sigma, \Sigma)$

// children

$$\mu(v, s) = 0$$

For  $w \in \delta(v)$

$$\mu(v, s) += \min_{t \in \Sigma} \{c(s, t) + \mu(w, t)\}$$

Traceback ( $T, v, \mu$ )

if  $v = r(T)$

$$\sigma(r(T)) = \arg \min_{s \in \Sigma} \{ \mu(r(T), s) \}$$

else let  $u$  be the parent of  $v$  and let  $s = \sigma(u)$

$$\sigma(v) = \arg \min_{t \in \Sigma} \{ c(s, t) + \mu(v, t) \}$$

For  $w \in \delta(v)$

Traceback ( $T, w, \mu$ )

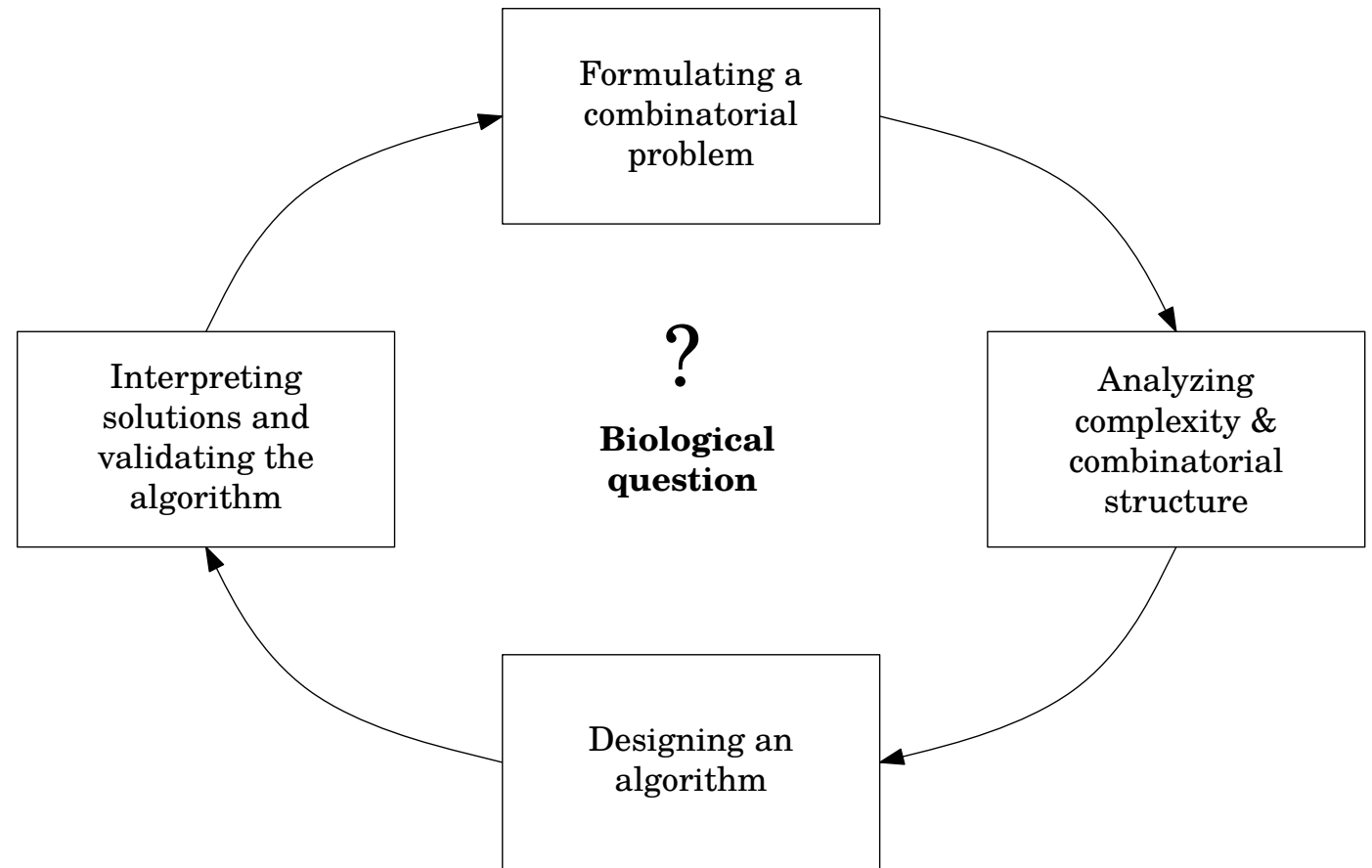
Let  $r(T)$  be the root vertex

# Outline

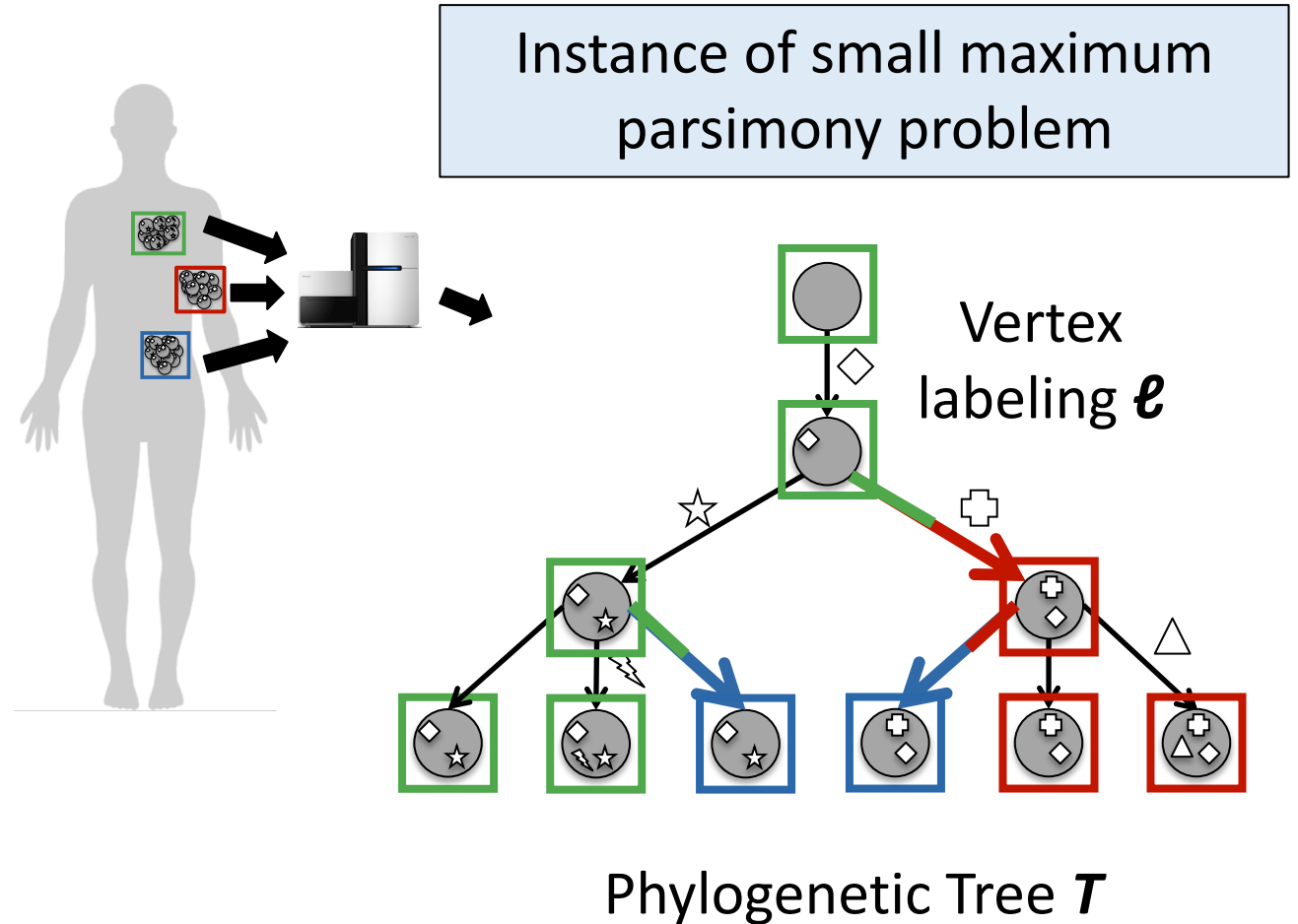
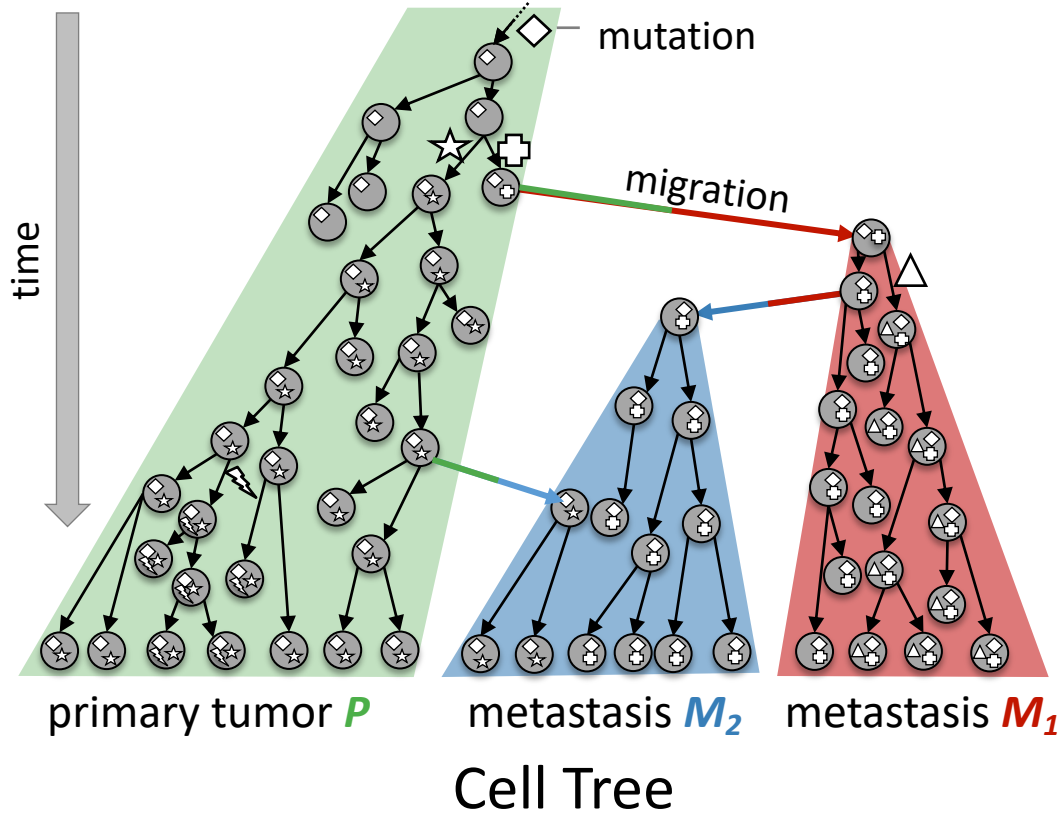
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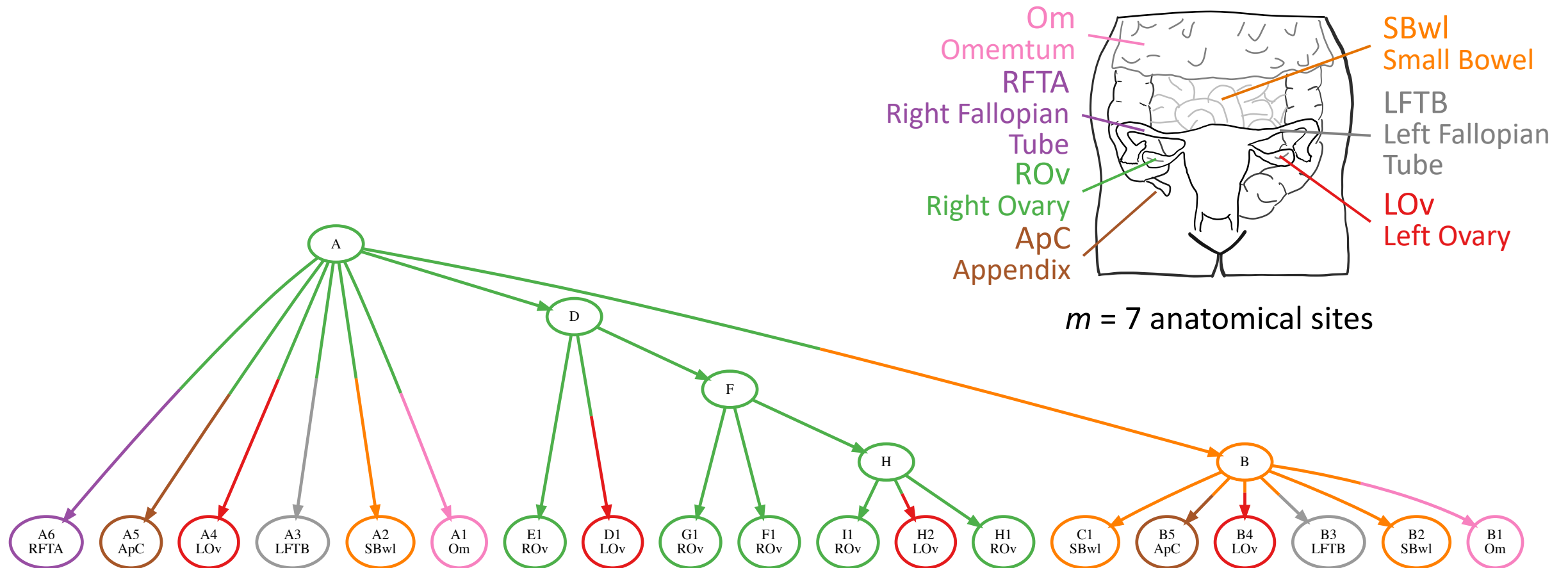


**Goal:** Given phylogenetic tree  $T$ , find *parsimonious* vertex labeling  $\ell$  with fewest migrations

# Minimum Migration Analysis in Ovarian Cancer

McPherson et al. (2016). Divergent modes of clonal spread and intraperitoneal mixing in high-grade serous ovarian cancer. *Nature Genetics*.

- Instance of the maximum parsimony small phylogeny problem [Fitch, 1971; Sankoff, 1975]

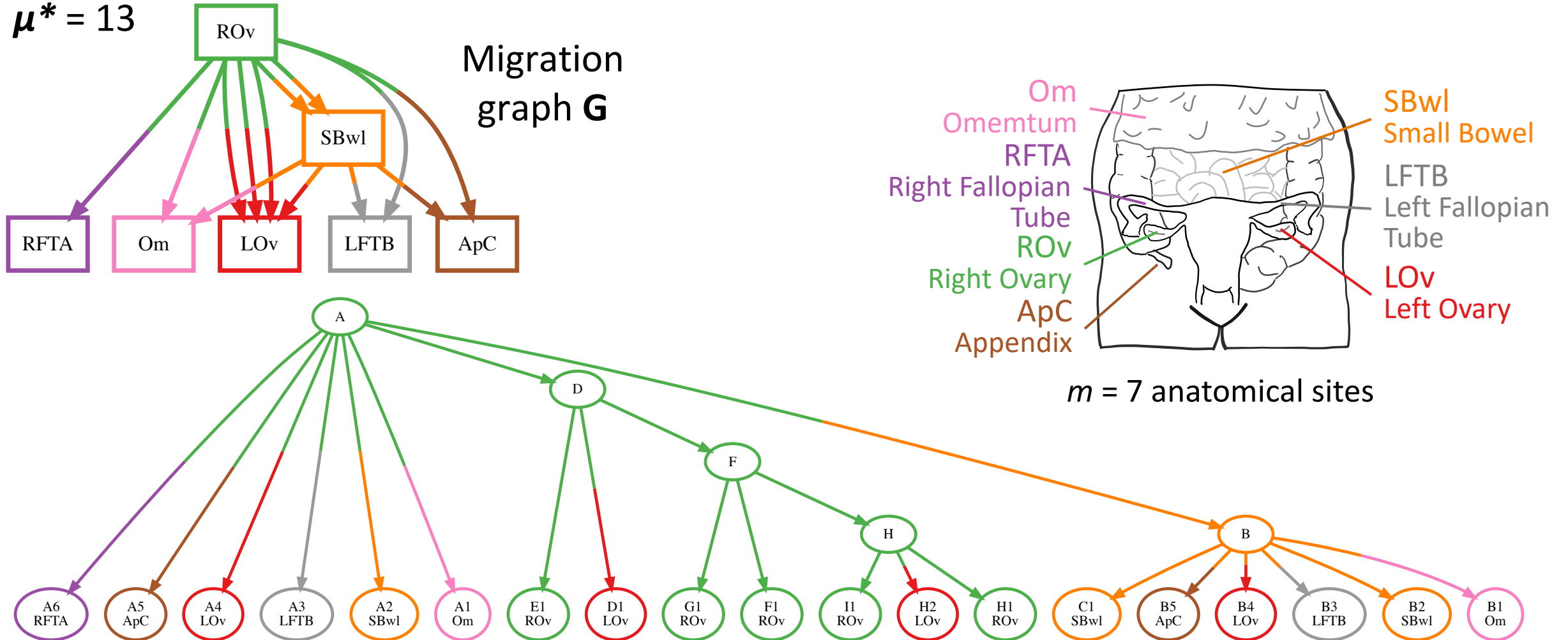


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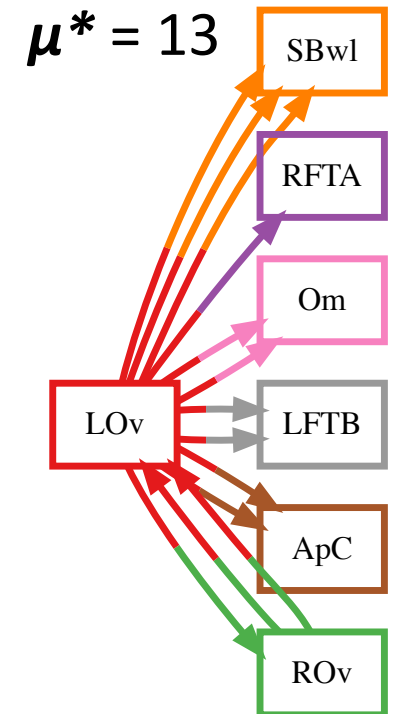
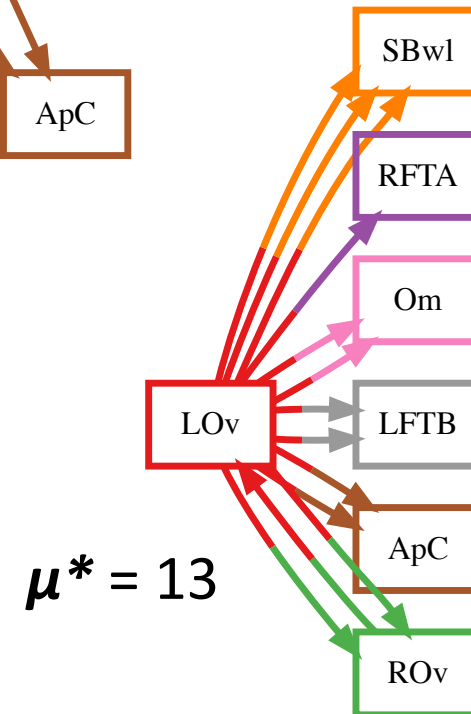
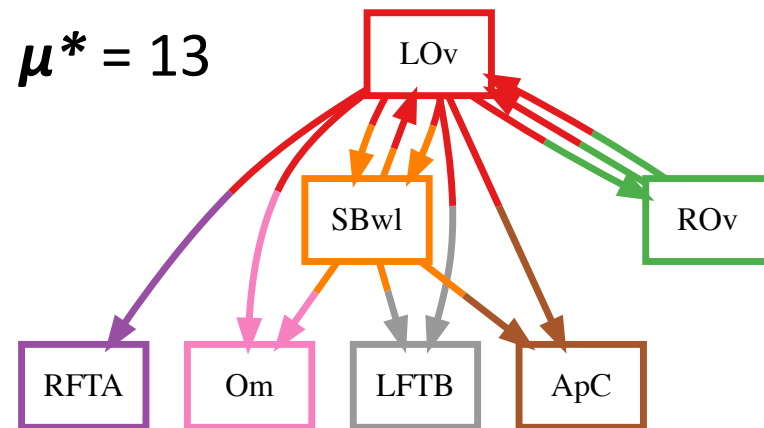
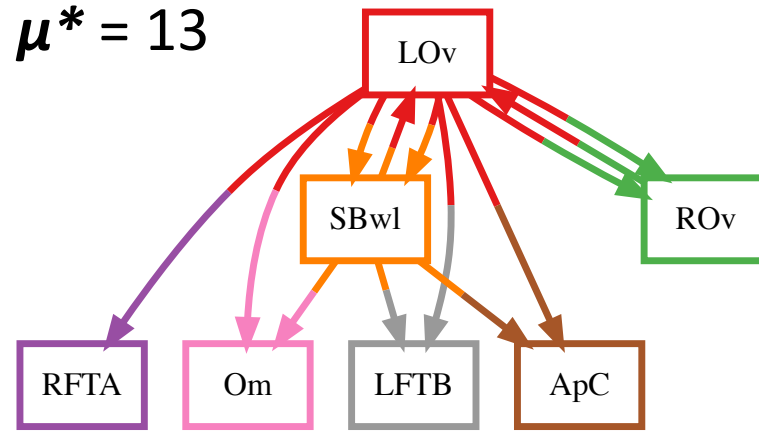
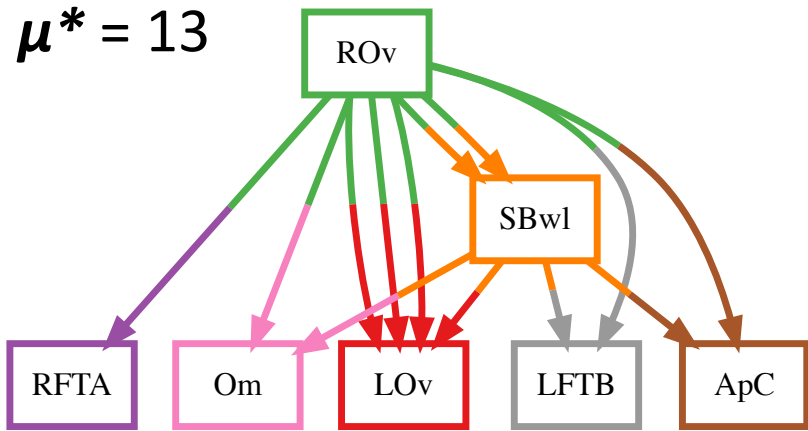
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$\mu^* = 13$



# Minimum Migration History is *Not* Unique

- Enumerate all minimum-migration vertex labelings in the backtrace step

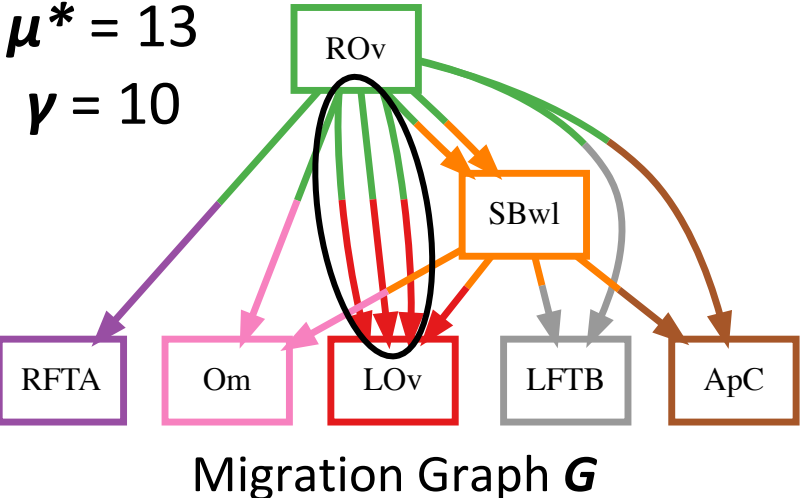


ApC	Appendix
LFTB	Left Fallopian Tube
LOv	Left Ovary
RFTA	Right Fallopian Tube
ROv	Right Ovary
SBwl	Small Bowel
Om	Omentum

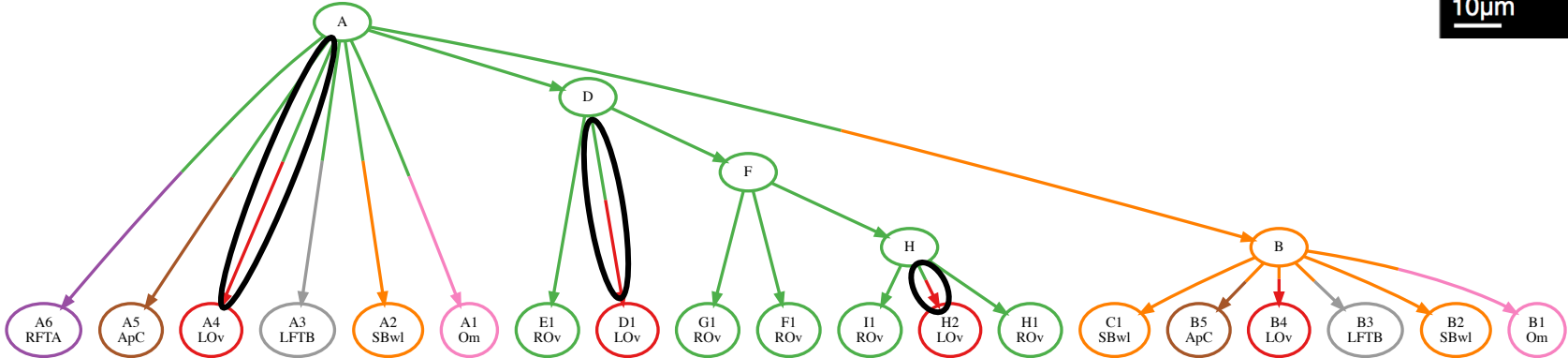
# Comigrations: Simultaneous Migrations of Multiple Clones

- Multiple tumor cells migrate simultaneously through the blood stream [Cheung et al., 2016]
- Second objective: number  $\nu$  of **comigrations** is the number of multi-edges in migration graph  $G^\dagger$

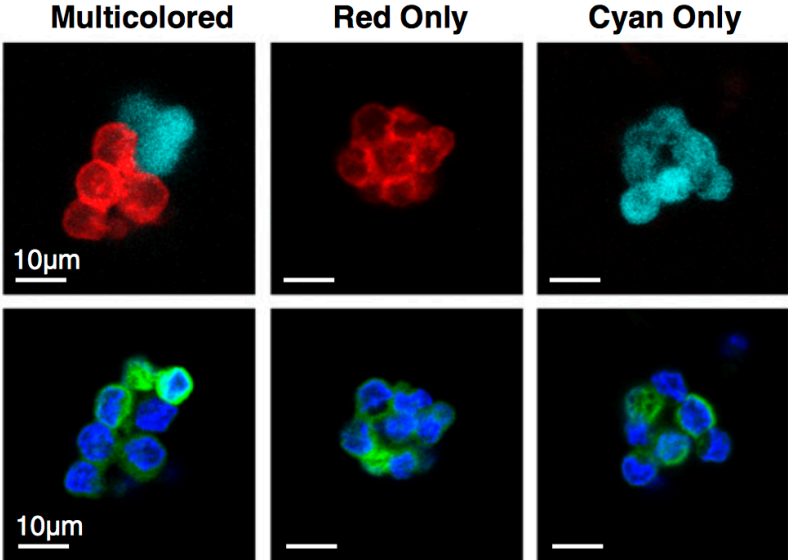
† Not necessarily true in the case of directed cycles



Clone Tree  $T$



F Circulating tumor cell clusters



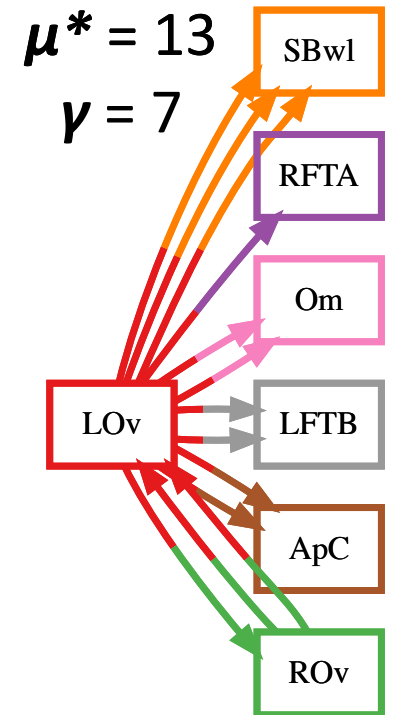
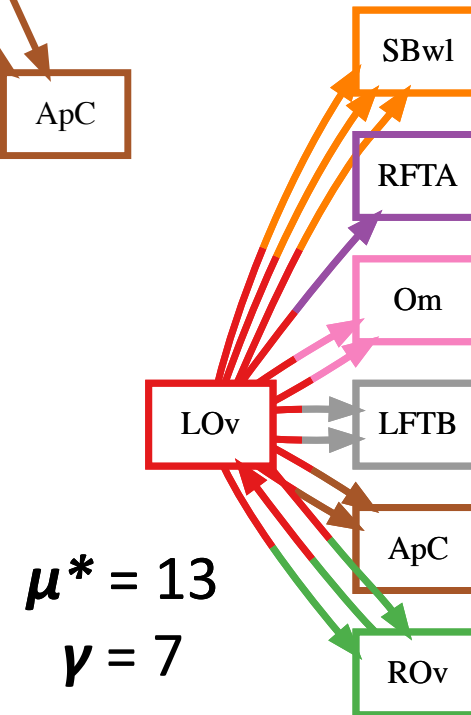
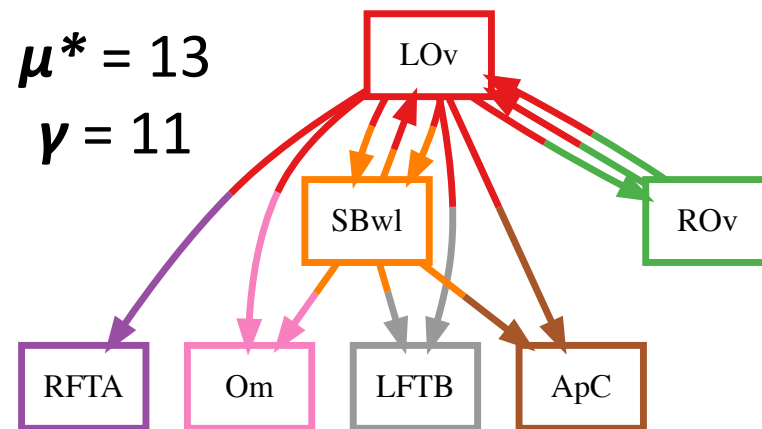
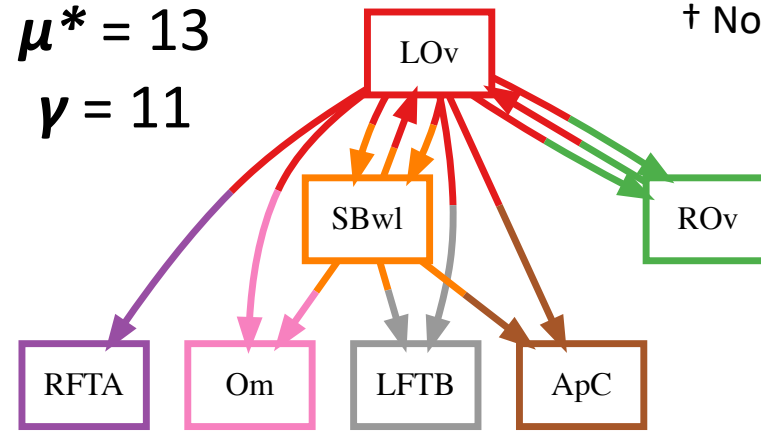
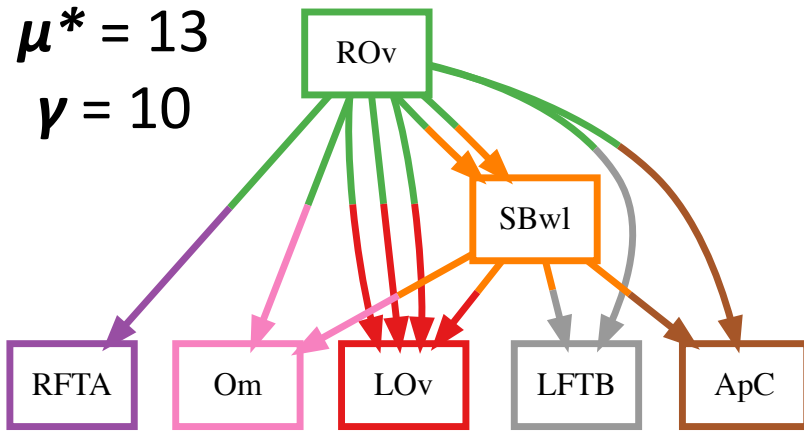
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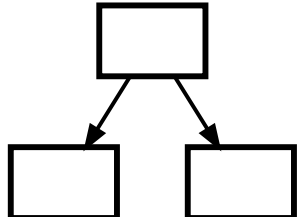
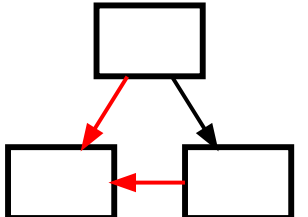
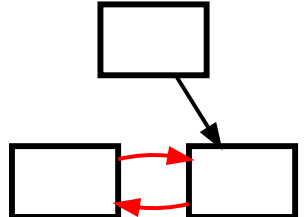
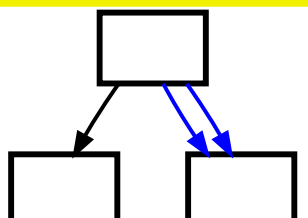
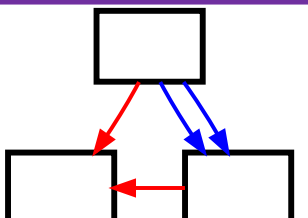
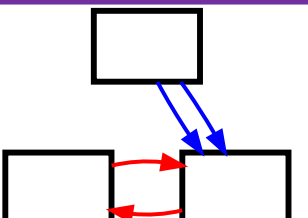
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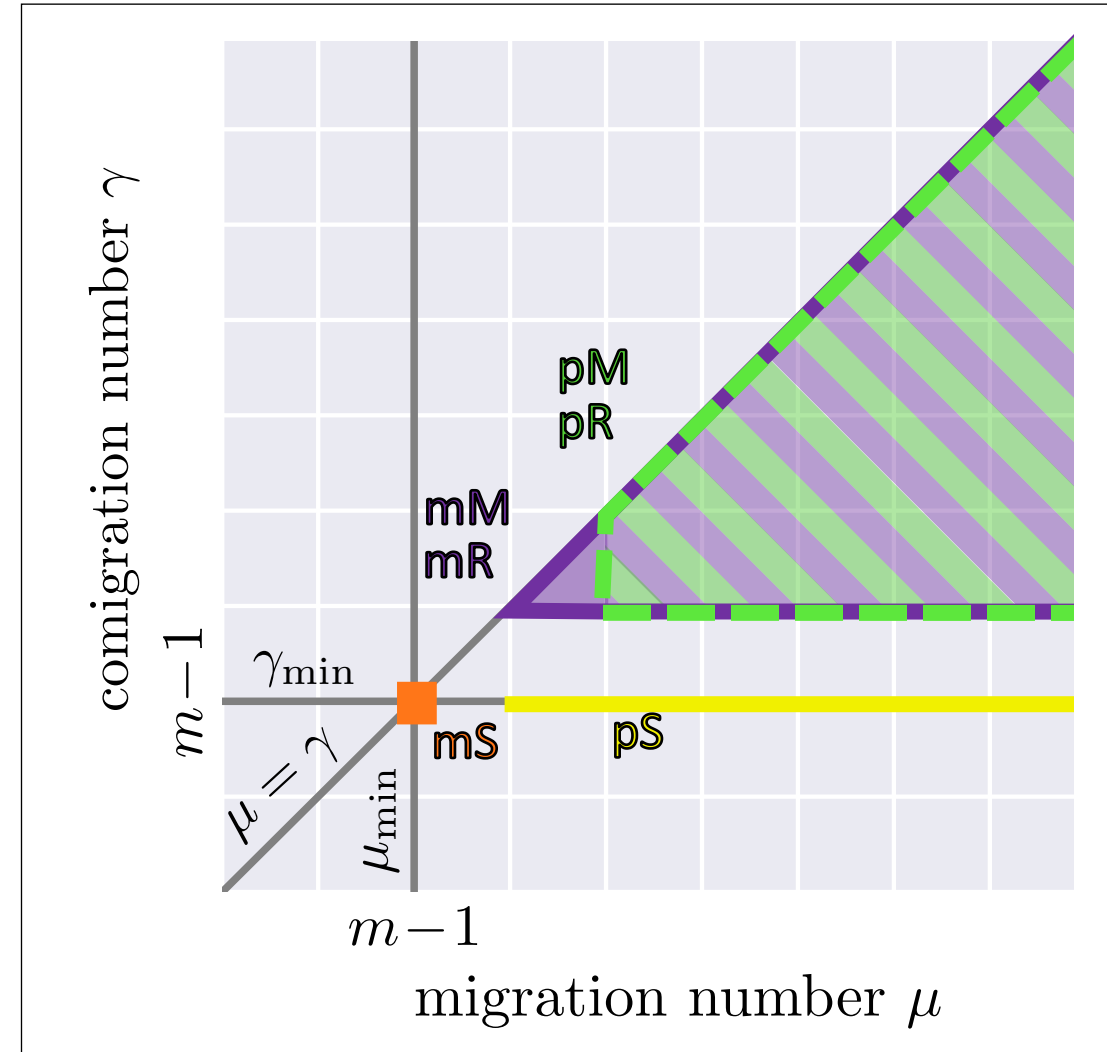
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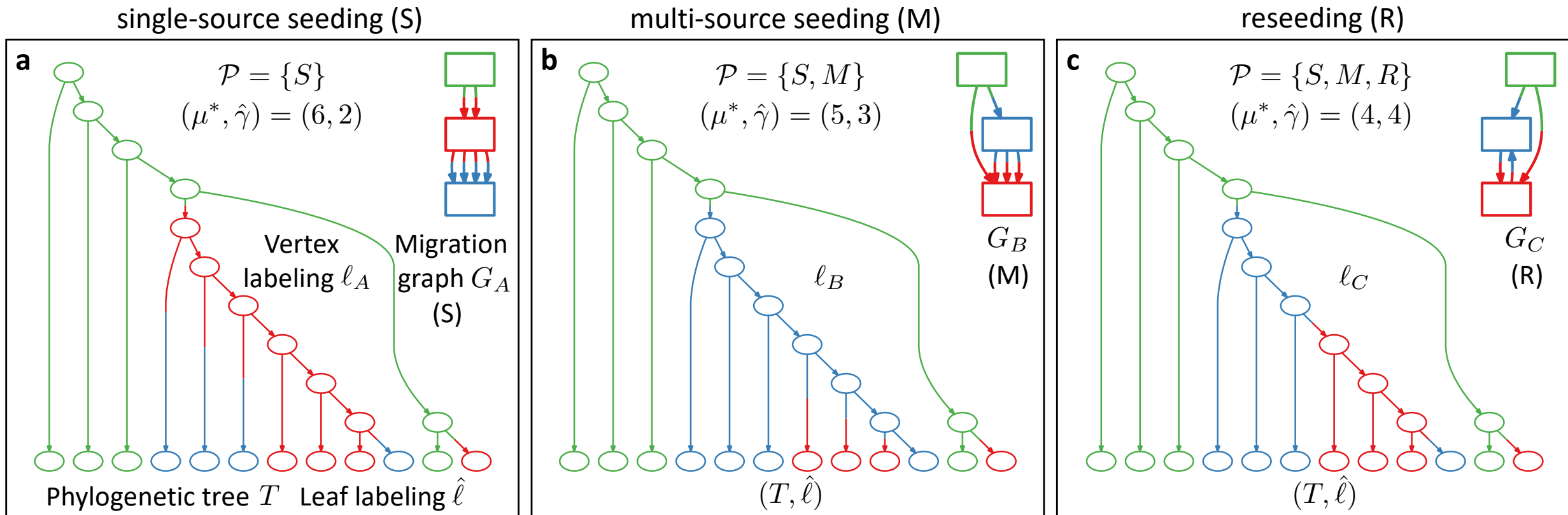
# Tradeoffs between Migrations, Comigrations and Migration Pattern

	single-source seeding (S)	multi-source seeding (M)	reseeding (R)
monoclonal (m)	 <p>tree</p>	 <p>directed acyclic graph</p>	 <p>directed graph</p>
polyclonal (p)	 <p>multi-tree</p>	 <p>directed acyclic multi-graph</p>	 <p>directed multi-graph</p>



# Constrained Multi-objective Optimization Problem

**Parsimonious Migration History (PMH):** Given a phylogenetic tree  $T$  and a set  $\mathcal{P} \subseteq \{S, M, R\}$  of allowed migration patterns, find vertex labeling  $\ell$  with minimum migration number  $\mu^*(T)$  and smallest comigration number  $\hat{\gamma}(T)$ .

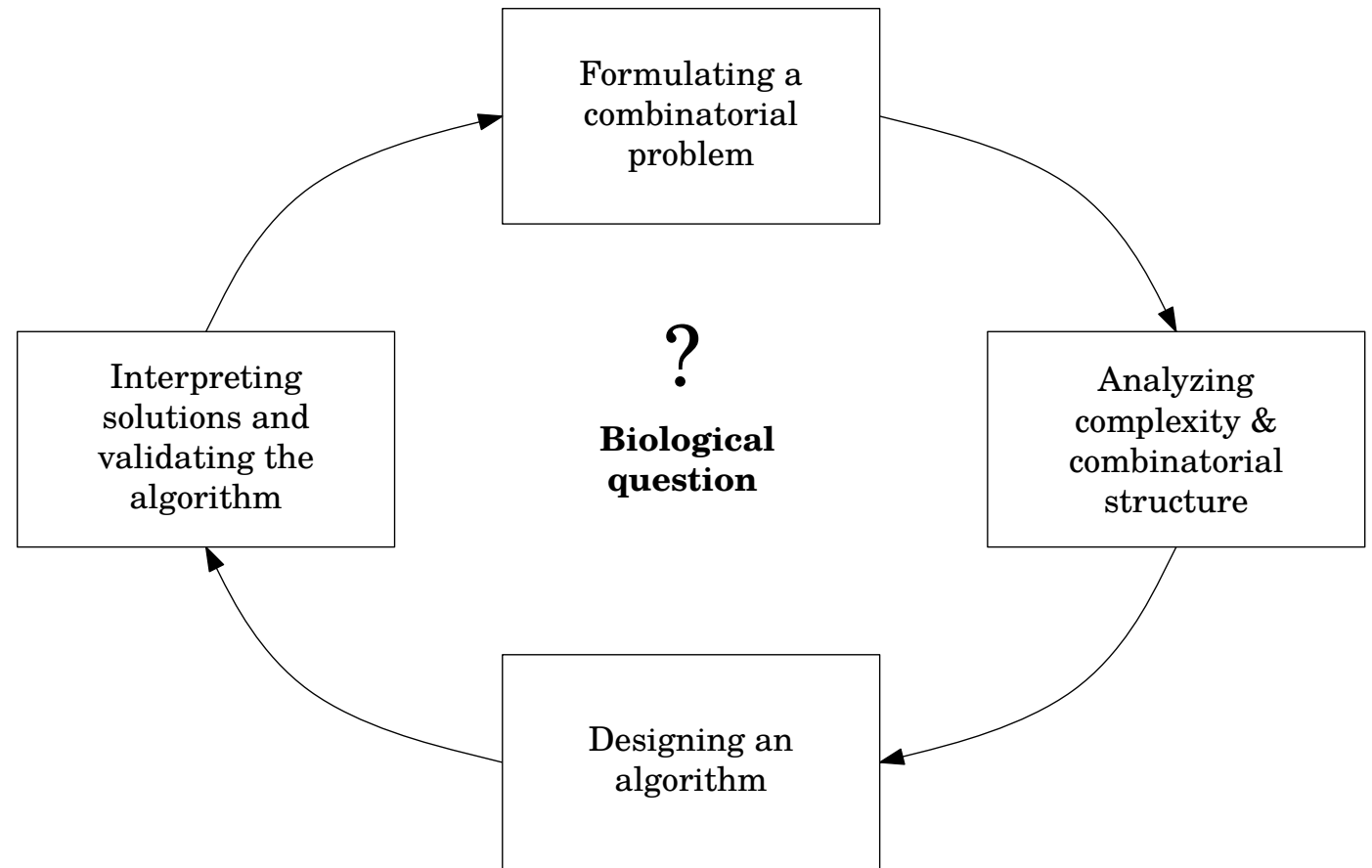


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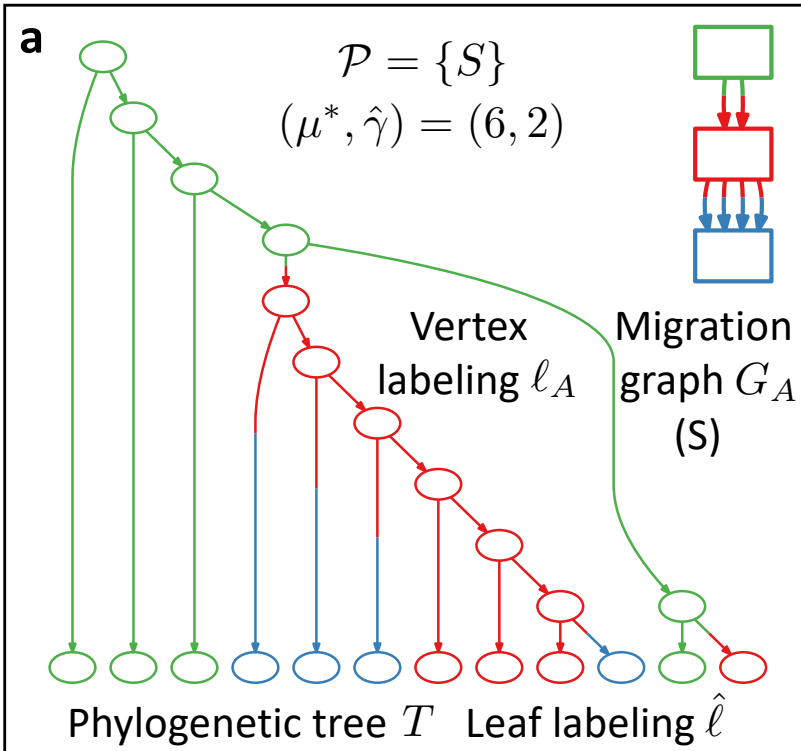
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# Results [El-Kebir, WABI 2018]

**Parsimonious Migration History (PMH):** Given a phylogenetic tree  $T$  and a set  $\mathcal{P} \subseteq \{S, M, R\}$  of allowed migration patterns, find vertex labeling  $\ell$  with minimum migration number  $\mu^*(T)$  and smallest comigration number  $\hat{\gamma}(T)$ .

single-source seeding (S)

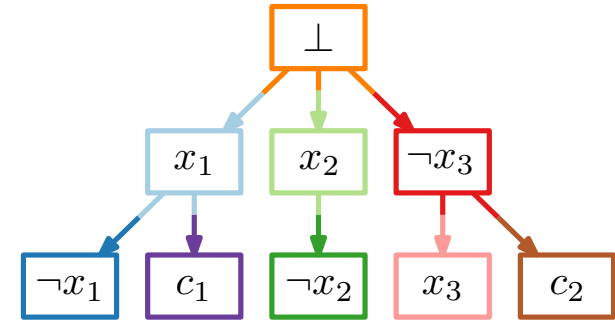


**Theorem 1:** PMH is NP-hard when  $\mathcal{P} = \{S\}$

**Theorem 2:** PMH is fixed parameter tractable in the number  $m$  of locations when  $\mathcal{P} = \{S\}$

# PMH is NP-hard when $\mathcal{P} = \{S\}$

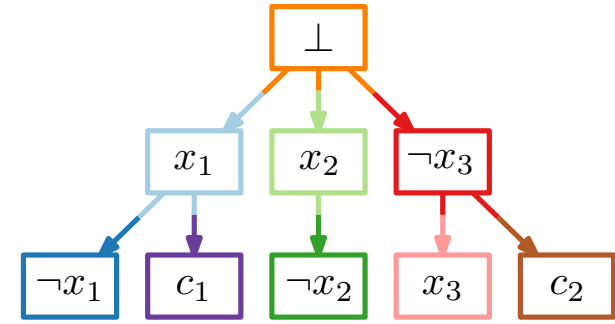
**3-SAT:** Given  $\varphi = \bigwedge_{i=1}^k (y_{i,1} \vee y_{i,2} \vee y_{i,3})$  with variables  $\{x_1, \dots, x_n\}$  and  $k$  clauses, find  $\phi : [n] \rightarrow \{0,1\}$  satisfying  $\varphi$



$$\Sigma = \{x_1, \dots, x_n, \neg x_1, \dots, \neg x_n, c_1, \dots, c_k, \perp\}$$

# PMH is NP-hard when $\mathcal{P} = \{S\}$

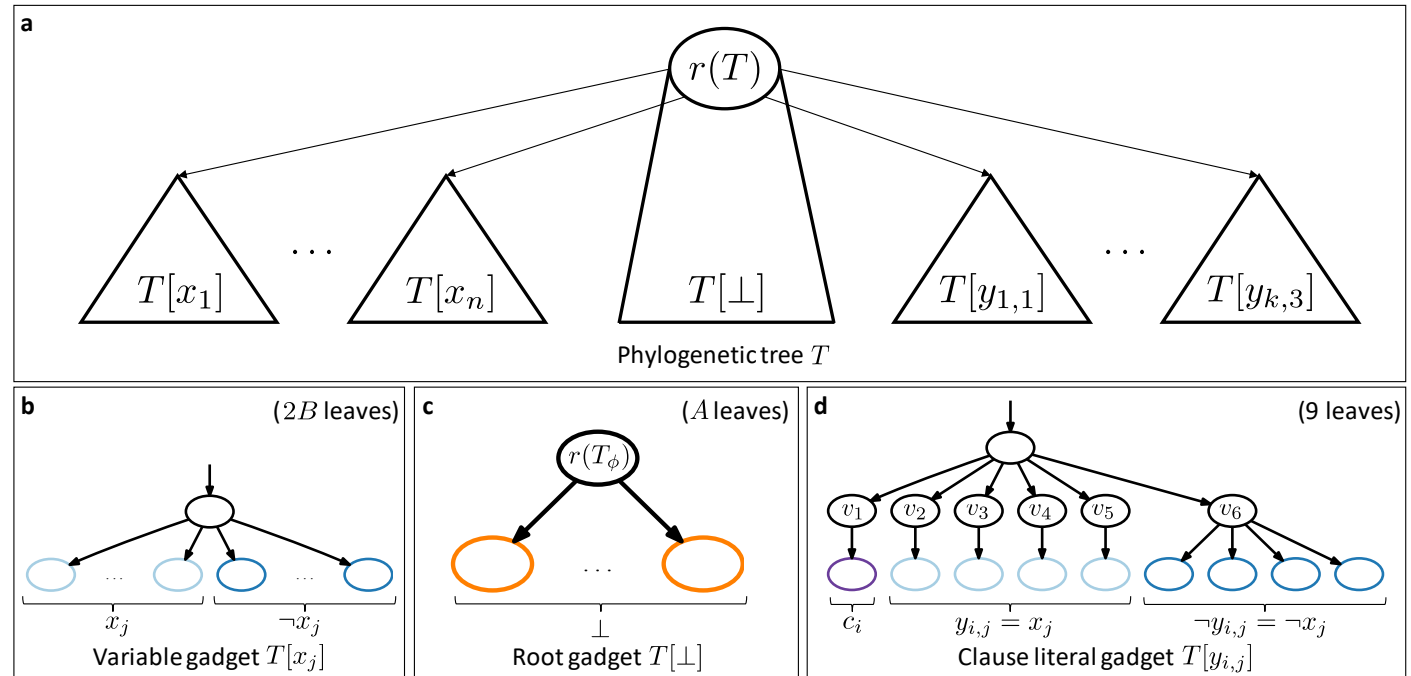
**3-SAT:** Given  $\varphi = \bigwedge_{i=1}^k (y_{i,1} \vee y_{i,2} \vee y_{i,3})$  with variables  $\{x_1, \dots, x_n\}$  and  $k$  clauses, find  $\phi : [n] \rightarrow \{0,1\}$  satisfying  $\varphi$



$$\Sigma = \{x_1, \dots, x_n, \neg x_1, \dots, \neg x_n, c_1, \dots, c_k, \perp\}$$

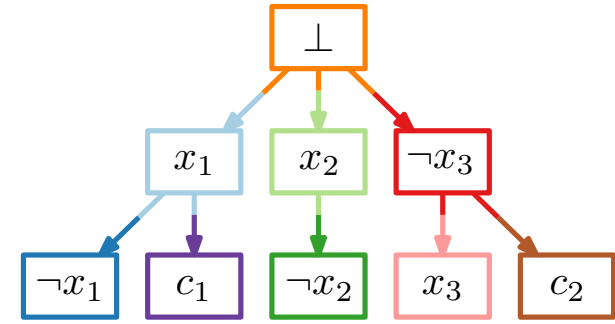
Three ideas:

1. Ensure that  $(x, \neg x) \in E(G)$  or  $(\neg x, x) \in E(G)$
2. Ensure that  $\ell^*(r(T)) = \perp$
3. Ensure that  $\varphi$  is satisfiable if and only if  $\ell^*$  encodes a satisfying truth assignment



# PMH is NP-hard when $\mathcal{P} = \{S\}$

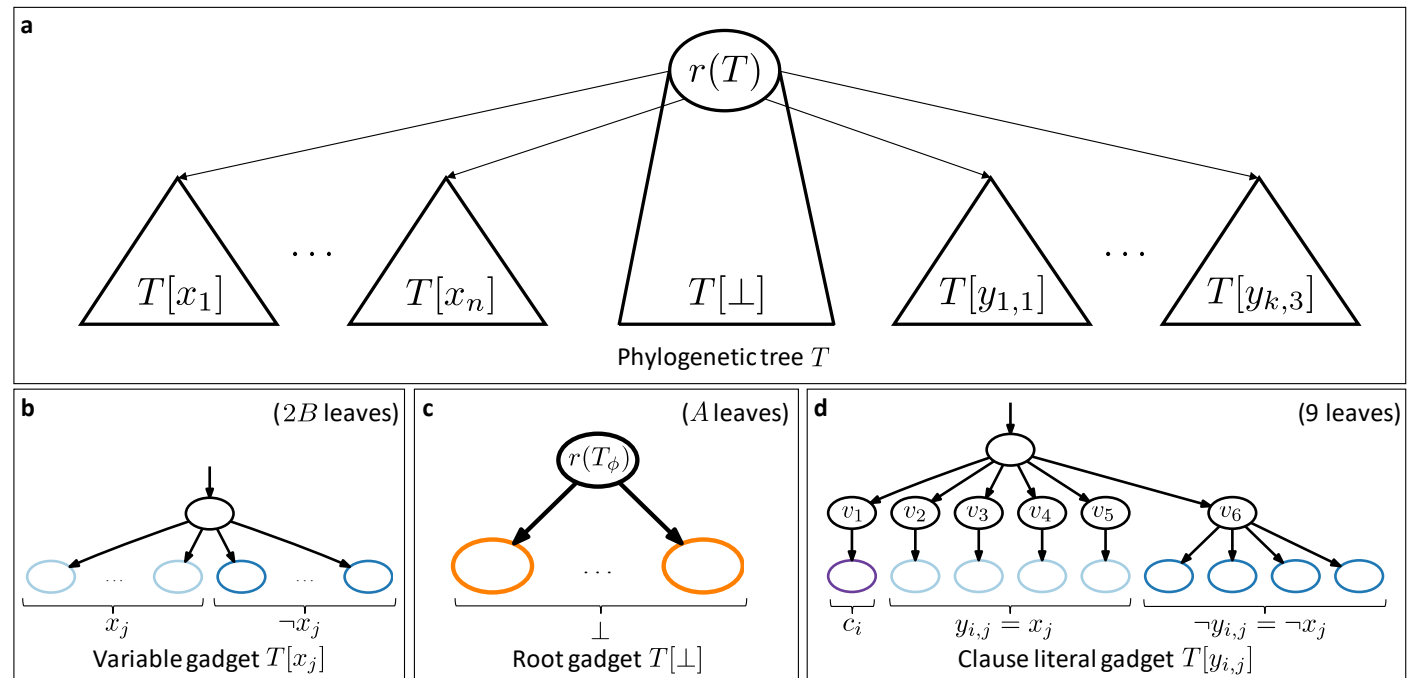
**3-SAT:** Given  $\varphi = \bigwedge_{i=1}^k (y_{i,1} \vee y_{i,2} \vee y_{i,3})$  with variables  $\{x_1, \dots, x_n\}$  and  $k$  clauses, find  $\phi : [n] \rightarrow \{0,1\}$  satisfying  $\varphi$



$$\Sigma = \{x_1, \dots, x_n, \neg x_1, \dots, \neg x_n, c_1, \dots, c_k, \perp\}$$

Three ideas:

1. Ensure that  $(x, \neg x) \in E(G)$  or  $(\neg x, x) \in E(G)$
2. Ensure that  $\ell^*(r(T)) = \perp$
3. Ensure that  $\varphi$  is satisfiable if and only if  $\ell^*$  encodes a satisfying truth assignment



**Lemma:** Let  $B > 10k + 1$  and  $A > 2Bn + 27k$ .

Then,  $\varphi$  is satisfiable if and only if  $\mu^*(T) = (B + 1)n + 25k$



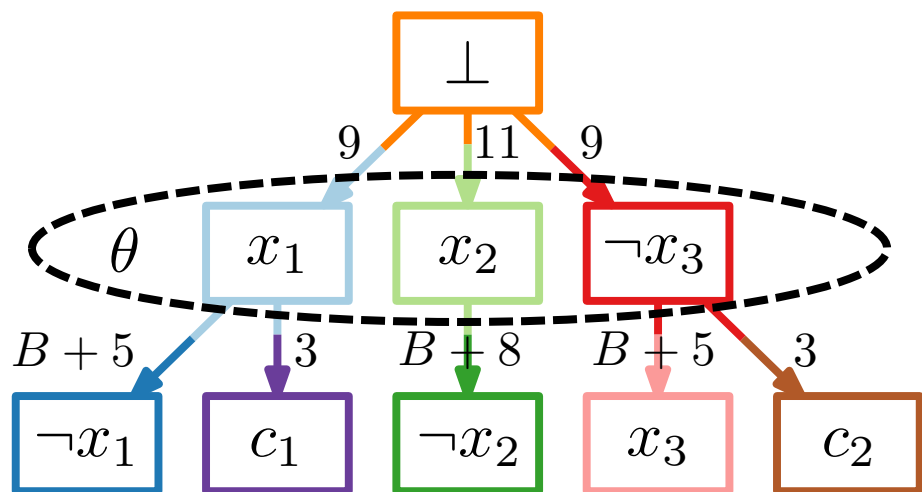
# PMH is NP-hard when $\mathcal{P} = \{S\}$

$$\varphi = (x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1, \neg x_2, \neg x_3)$$

$$k = 2, n = 3$$

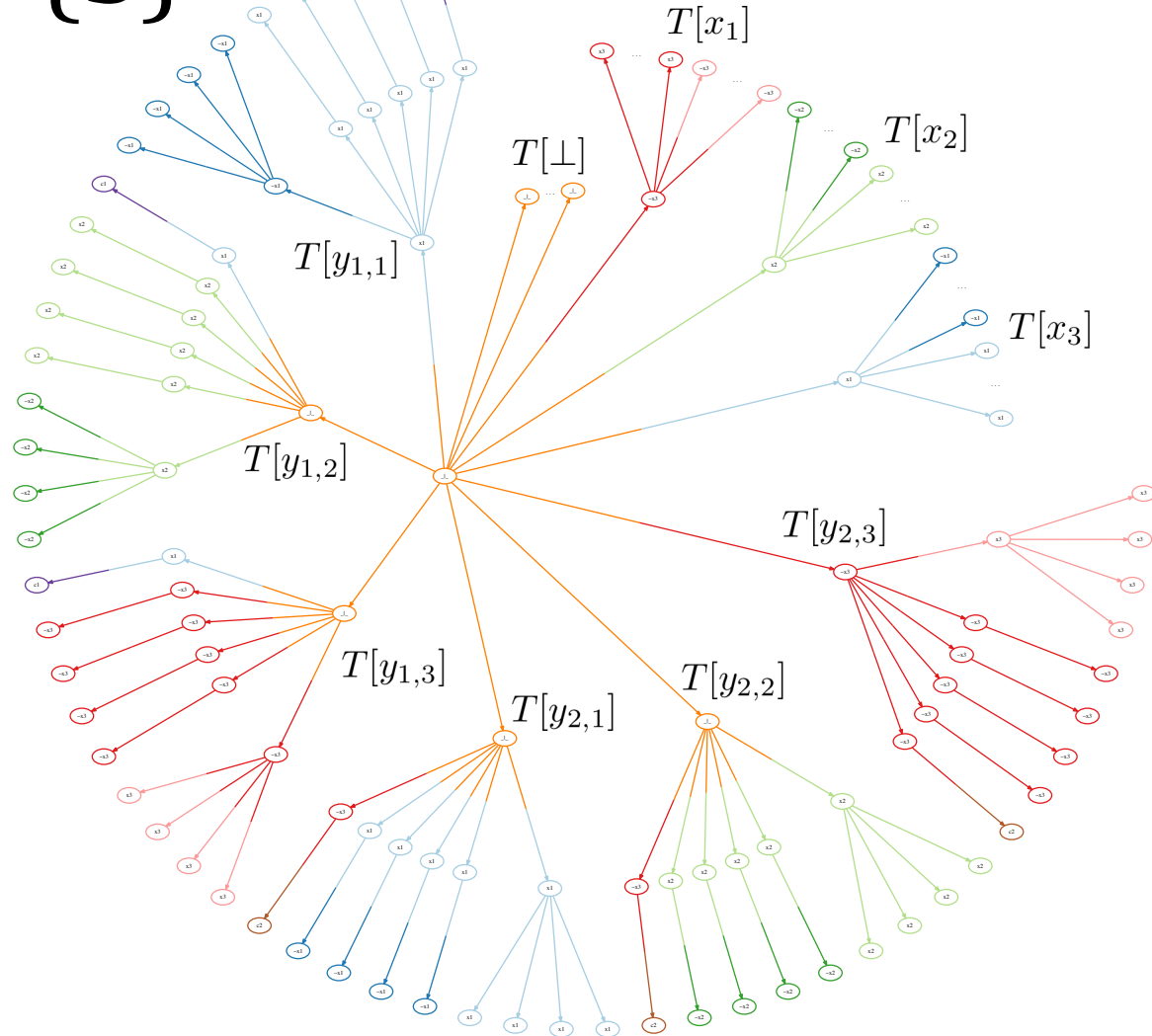
$$B = 10k + 2 = 22$$

$$A = 2Bn + 27k + 1 = 187$$



$$\Sigma = \{x_1, x_2, x_3, \neg x_1, \neg x_2, \neg x_3, c_1, c_2, \perp\}$$

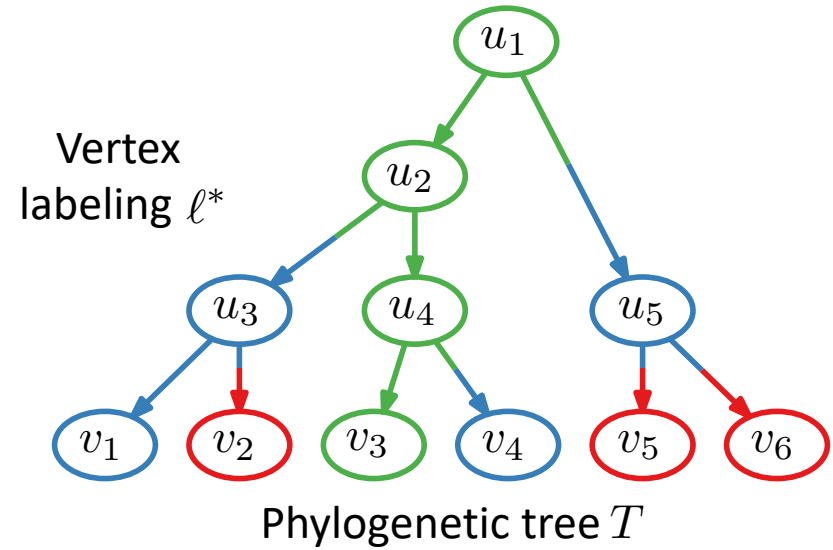
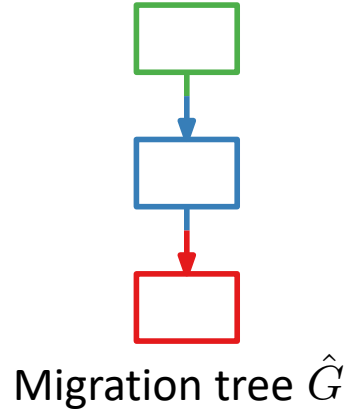
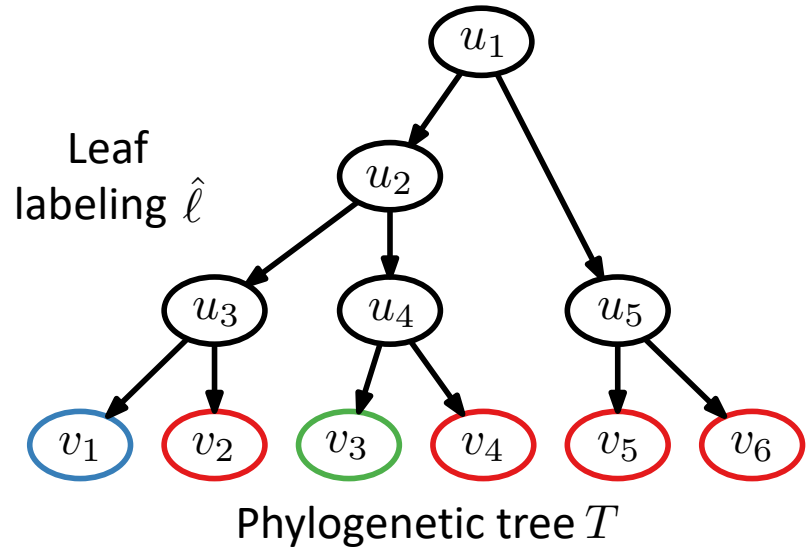
$$\begin{aligned} \mu^*(T) &= (B + 1)n + 25k \\ &= 23 * 3 + 50 * 2 = 119 \end{aligned}$$



**Lemma:** Let  $B > 10k + 1$  and  $A > 2Bn + 27k$ .

Then,  $\varphi$  is satisfiable if and only if  $\mu^*(T) = (B + 1)n + 25k$

PMH is FPT in number  $m$  of locations when  $\mathcal{P} = \{S\}$



**Lemma:** If there exists labeling  $\ell$  consistent with  $\hat{G}$  then

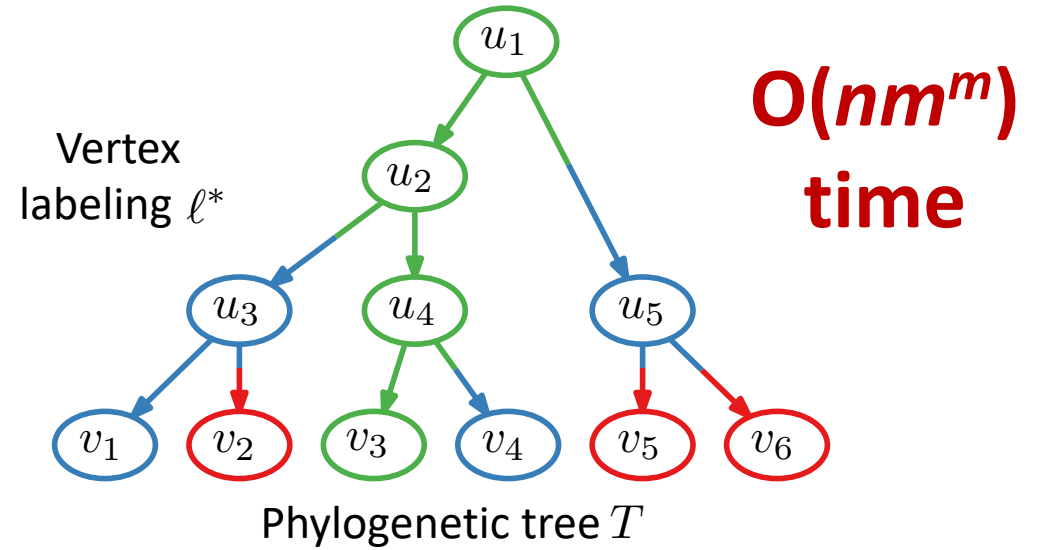
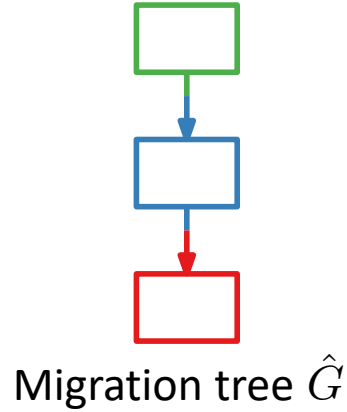
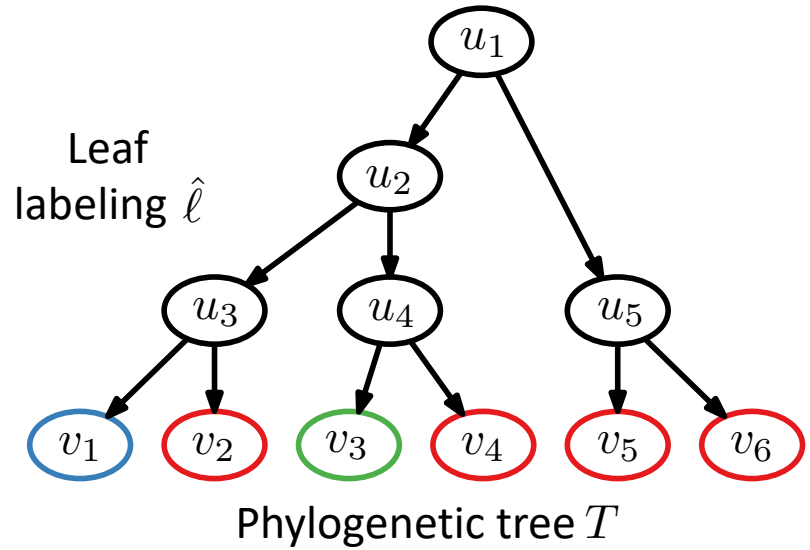
$$d_T(u, v) \geq d_{\hat{G}}(\text{lca}_{\hat{G}}(u), \hat{\ell}(v)) \quad \forall u, v \in V(T) \text{ such that } u \preceq_T v. \quad (1)$$

$$\ell^*(v) = \begin{cases} \text{LCA}_{\hat{G}}(r(T)), & \text{if } v = r(T), \\ \sigma(\ell^*(\pi(v)), \text{LCA}_{\hat{G}}(v)), & \text{if } v \neq r(T), \end{cases}$$

where  $\sigma(s, t) = s$  if  $s = t$  and otherwise  $\sigma(s, t)$  is the unique child of  $s$  that lies on the path from  $s$  to  $t$  in  $\hat{G}$ .

**Lemma:** If (1) holds then  $\ell^*$  is a minimum migration labeling consistent with  $\hat{G}$ .

PMH is FPT in number  $m$  of locations when  $\mathcal{P} = \{S\}$



**Lemma:** If there exists labeling  $\ell$  consistent with  $\hat{G}$  then

$$d_T(u, v) \geq d_{\hat{G}}(\text{lca}_{\hat{G}}(u), \hat{\ell}(v)) \quad \forall u, v \in V(T) \text{ such that } u \preceq_T v. \quad (1)$$

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where  $\sigma(s, t) = s$  if  $s = t$  and otherwise  $\sigma(s, t)$  is the unique child of  $s$  that lies on the path from  $s$  to  $t$  in  $\hat{G}$ .

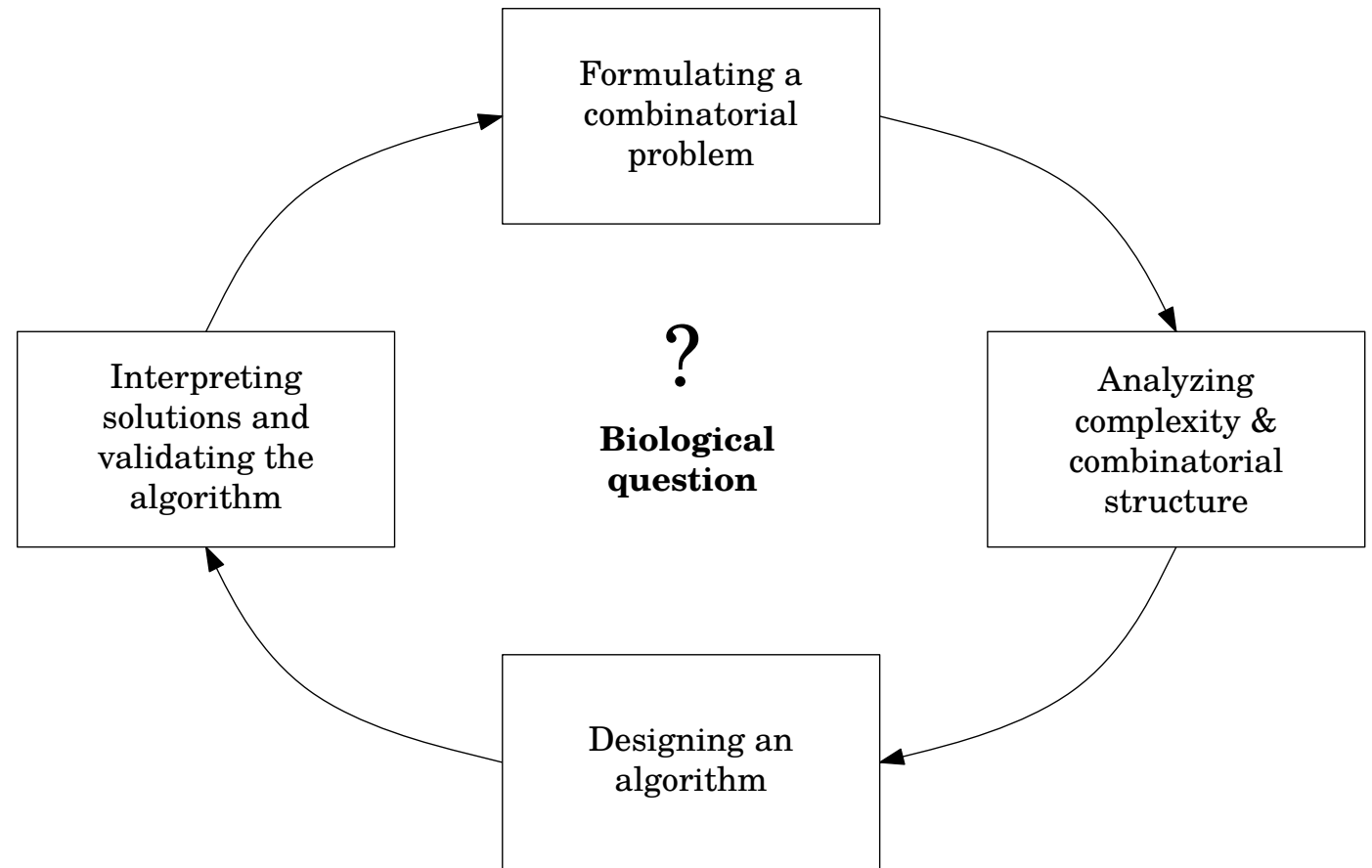
**Lemma:** If (1) holds then  $\ell^*$  is a minimum migration labeling consistent with  $\hat{G}$ .

# Outline

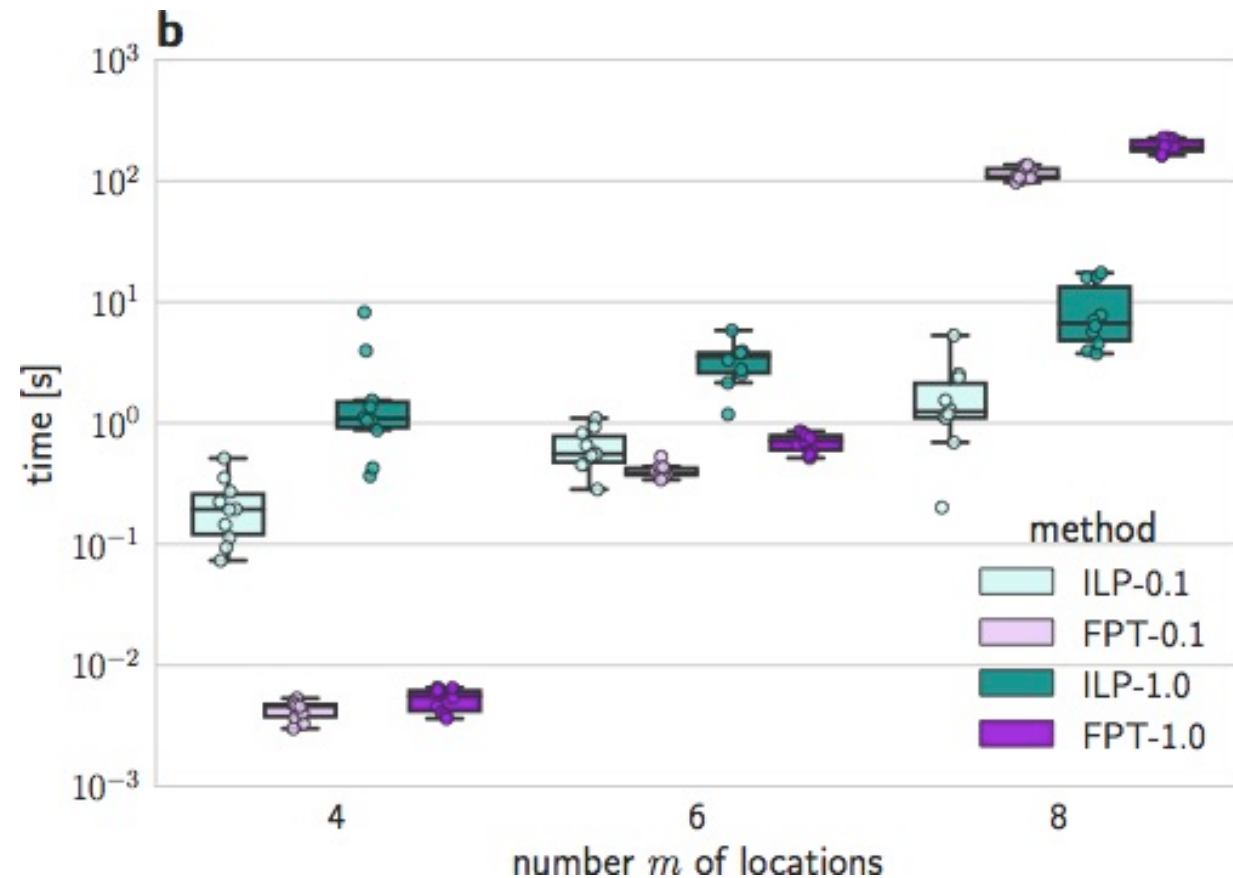
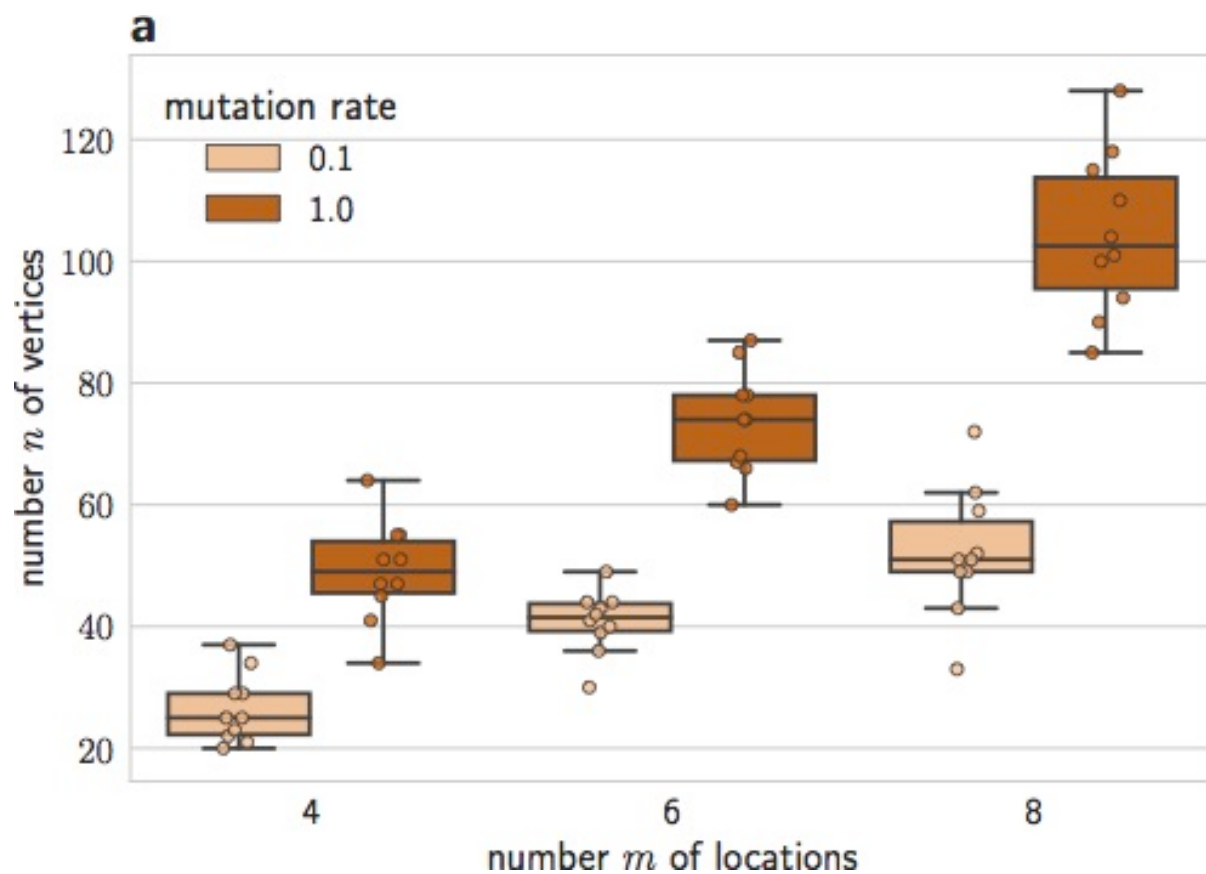
- Metastasis
- Maximum parsimony
- Problem statement
- Complexity
- Algorithm & results
- Problem variants

## Reading:

- **M. El-Kebir**, G. Satas and B.J. Raphael. Inferring parsimonious migration histories for metastatic cancers. [Nature Genetics, 50:718-726, 2018.](#)
- **M. El-Kebir**<sup>†</sup>. [Parsimonious Migration History Problem: Complexity and Algorithms.](#) WABI 2018, Helsinki, Finland, August 20-22, 2018.



# Simulations



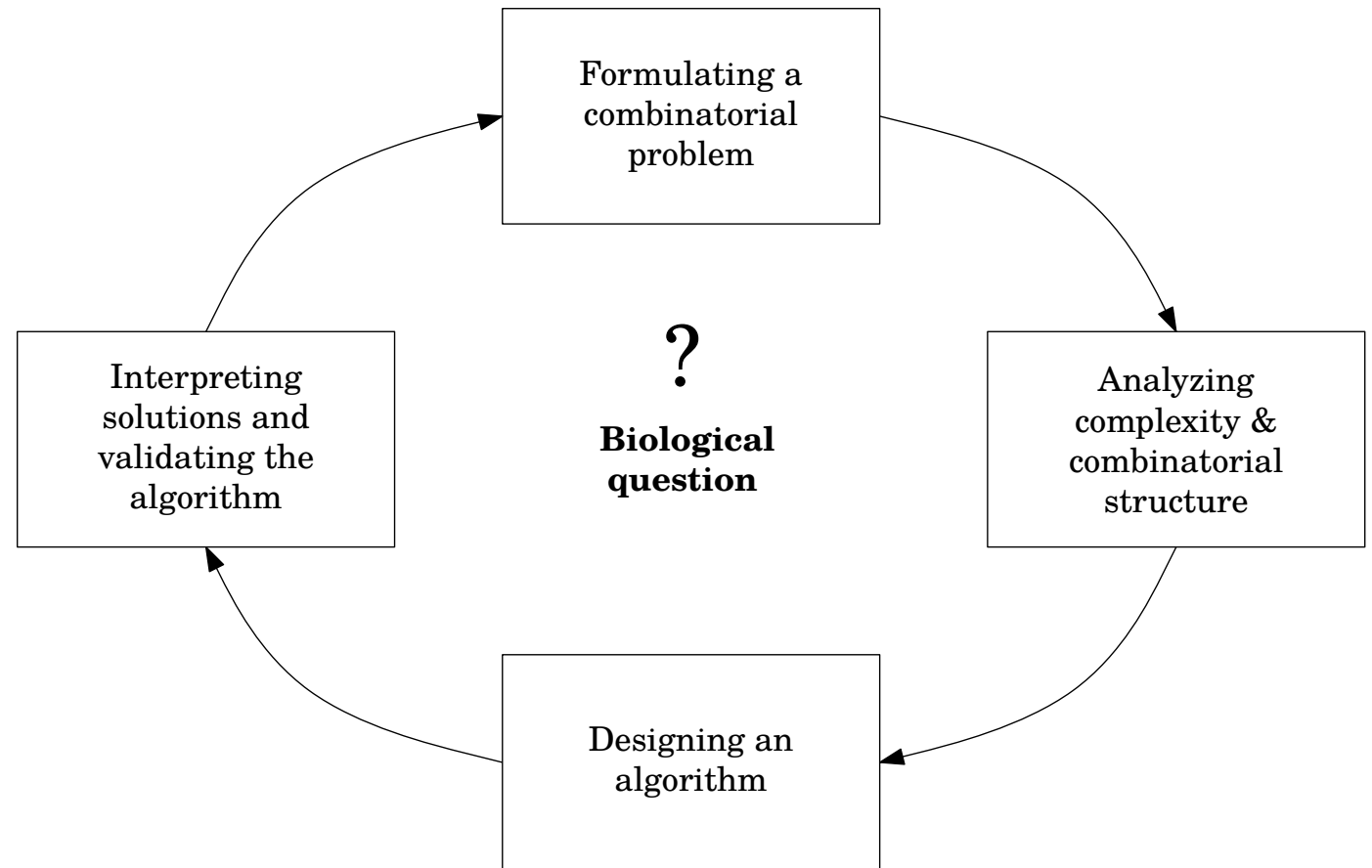
Available on: <https://github.com/elkebir-group/PMH-S>

# Outline

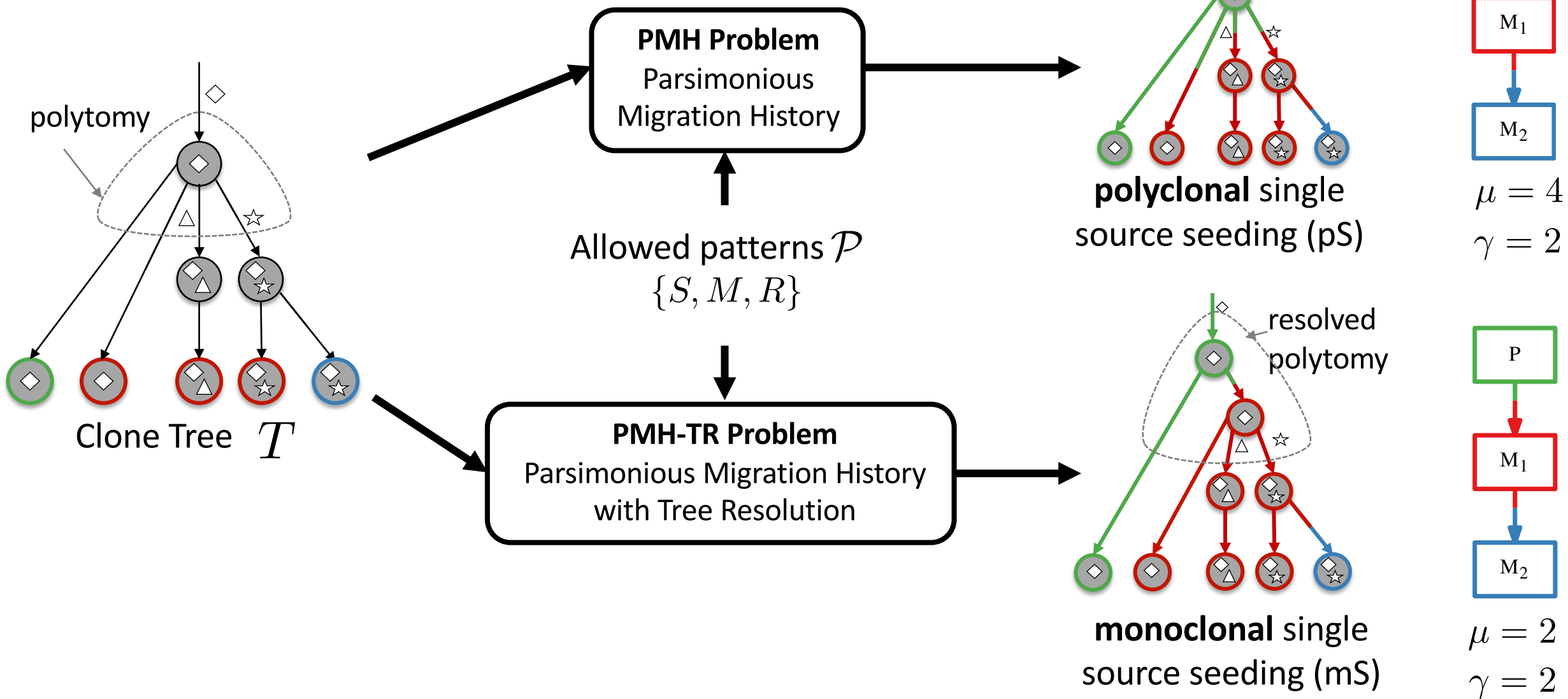
- Metastasis
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## Reading:

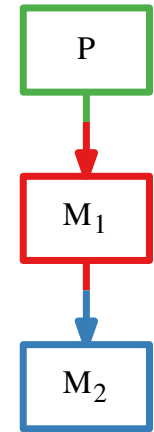
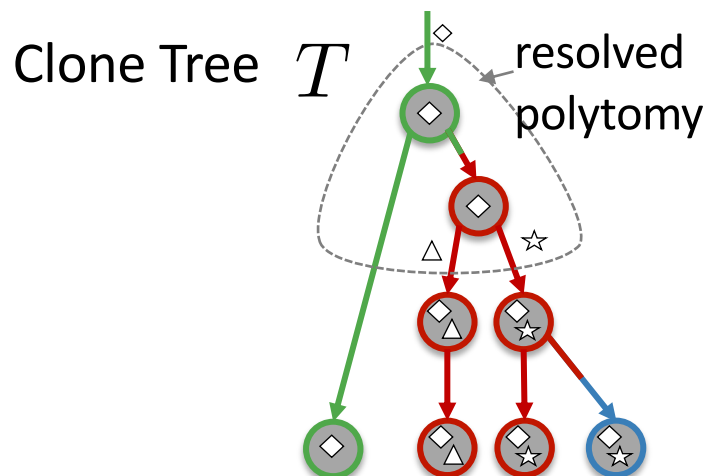
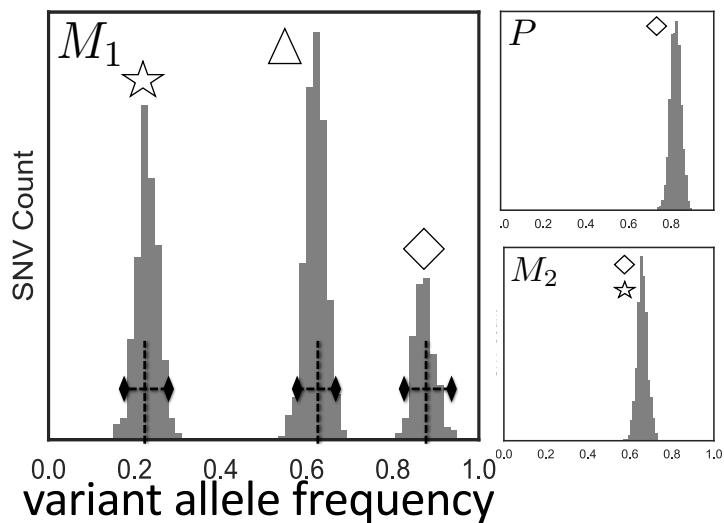
- **M. El-Kebir**, G. Satas and B.J. Raphael. Inferring parsimonious migration histories for metastatic cancers. [Nature Genetics, 50:718-726, 2018.](#)
- **M. El-Kebir**<sup>†</sup>. [Parsimonious Migration History Problem: Complexity and Algorithms.](#) WABI 2018, Helsinki, Finland, August 20-22, 2018.



# Resolving Clone Tree Ambiguities



# Resolving Clone Tree Ambiguities



$$\mu = 2$$

$$\gamma = 2$$

**monoclonal** single source seeding (mS)

confidence intervals

$$F^- = \begin{matrix} & \diamond & \triangle & \star \\ P & \begin{bmatrix} 0.75 & 0 & 0 \end{bmatrix} \\ M_1 & \begin{bmatrix} 0.82 & 0.15 & 0.56 \end{bmatrix} \\ M_2 & \begin{bmatrix} 0.66 & 0 & 0.68 \end{bmatrix} \end{matrix}$$

$$F^+ = \begin{matrix} & \diamond & \triangle & \star \\ P & \begin{bmatrix} 0.86 & 0 & 0 \end{bmatrix} \\ M_1 & \begin{bmatrix} 0.92 & 0.30 & 0.68 \end{bmatrix} \\ M_2 & \begin{bmatrix} 0.78 & 0 & 0.77 \end{bmatrix} \end{matrix}$$

$$\begin{matrix} & \diamond & \triangle & \star \\ P & \begin{bmatrix} 0.82 & 0 & 0 \end{bmatrix} \\ M_1 & \begin{bmatrix} 0.88 & 0.24 & 0.64 \end{bmatrix} \\ M_2 & \begin{bmatrix} 0.73 & 0 & 0.73 \end{bmatrix} \end{matrix} = \begin{matrix} & \diamond & \triangle & \star \\ P & \begin{bmatrix} 0.82 & 0 & 0 \end{bmatrix} \\ M_1 & \begin{bmatrix} 0 & 0.24 & 0.64 \end{bmatrix} \\ M_2 & \begin{bmatrix} 0 & 0 & 0.73 \end{bmatrix} \end{matrix} \begin{matrix} \diamond & \triangle & \star \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

$\hat{F} \qquad U \qquad B$

**PMH-TI Problem**  
Parsimonious Migration History with Tree Inference

$\mathcal{P}$

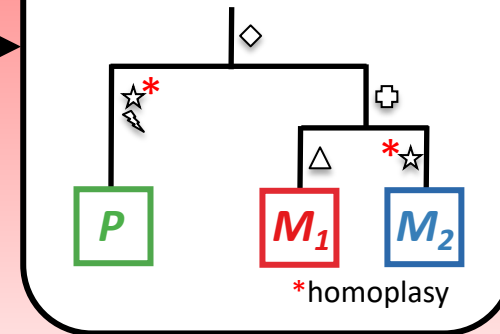


# Standard Phylogenetic Techniques

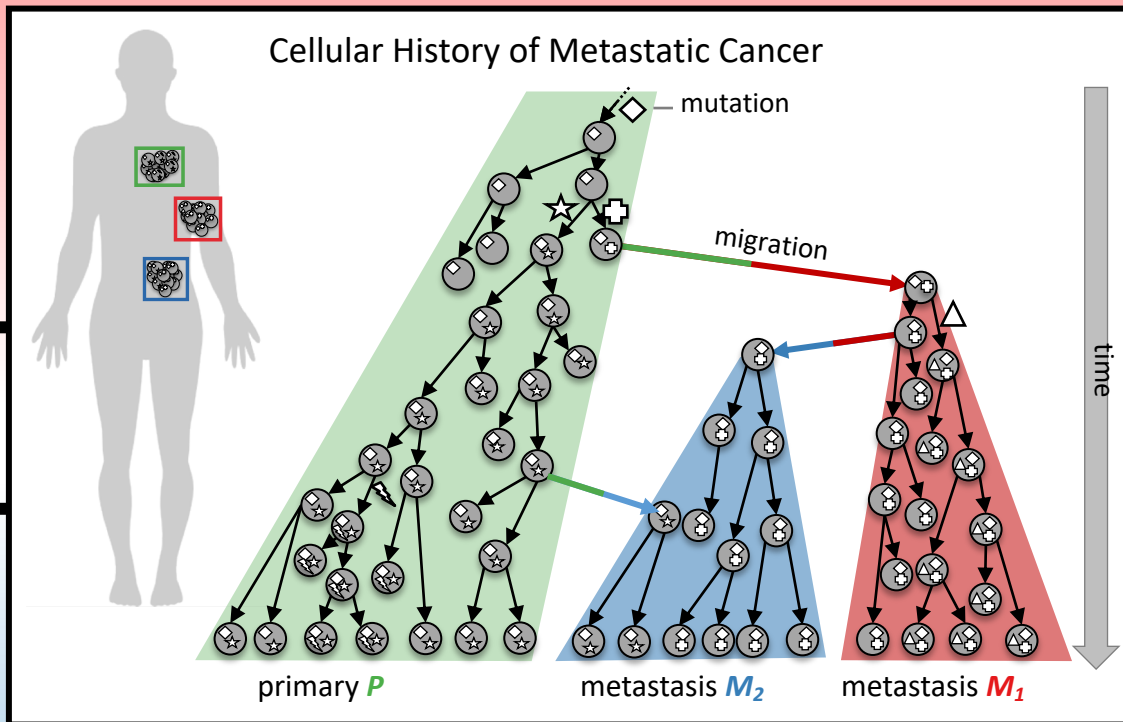
## Mutation Matrix



## Sample Tree



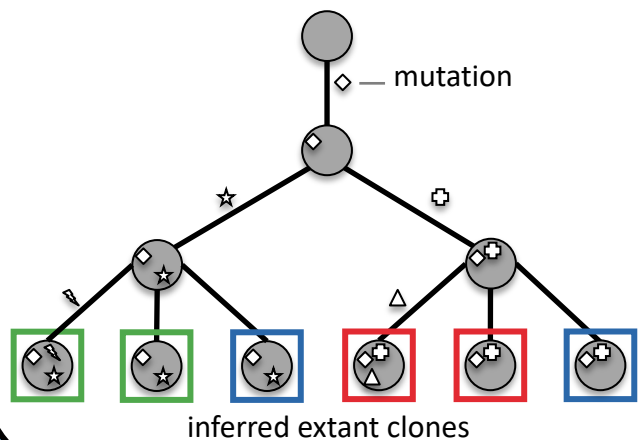
## Cellular History of Metastatic Cancer



Sequencing and Mutation Calling

## Tumor Phylogenetic Techniques

### Clone Tree



Cell Division and Mutation History

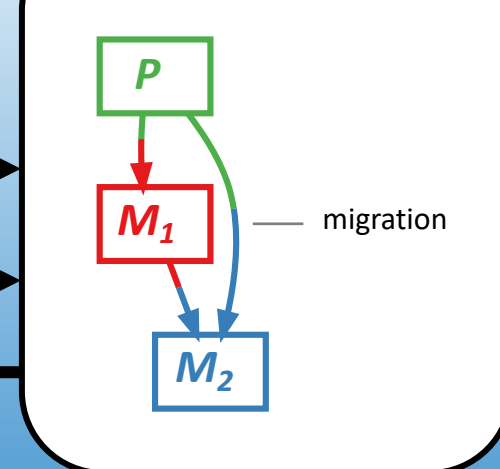
**MACHINA**

Cell Migration History

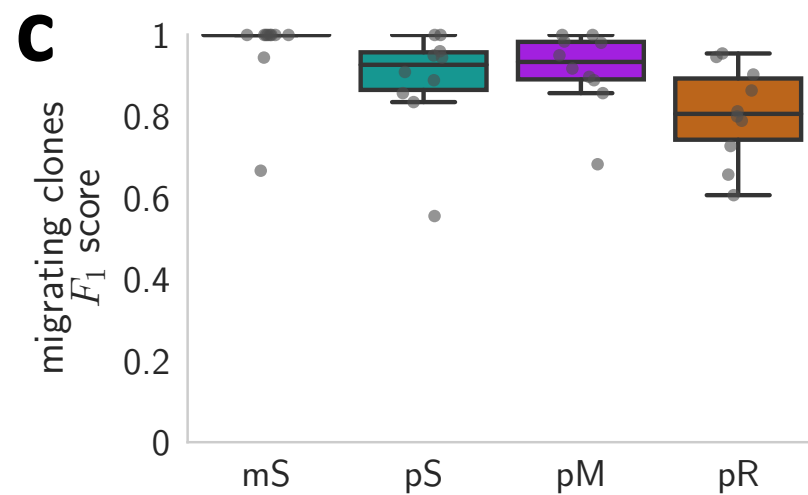
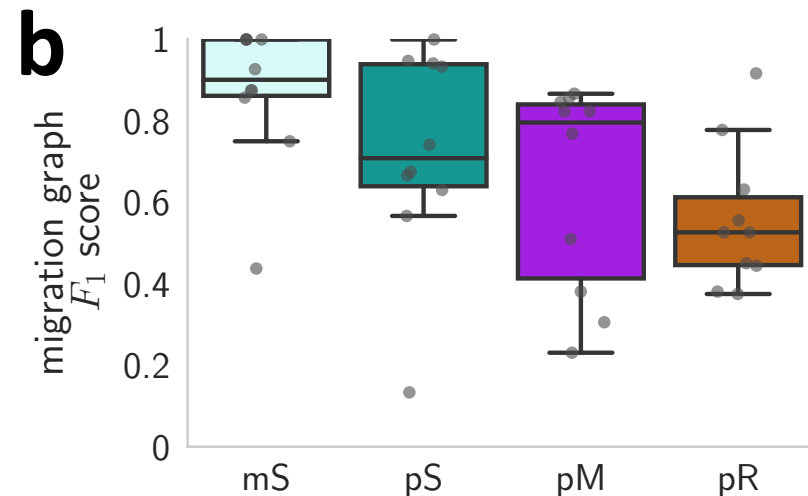
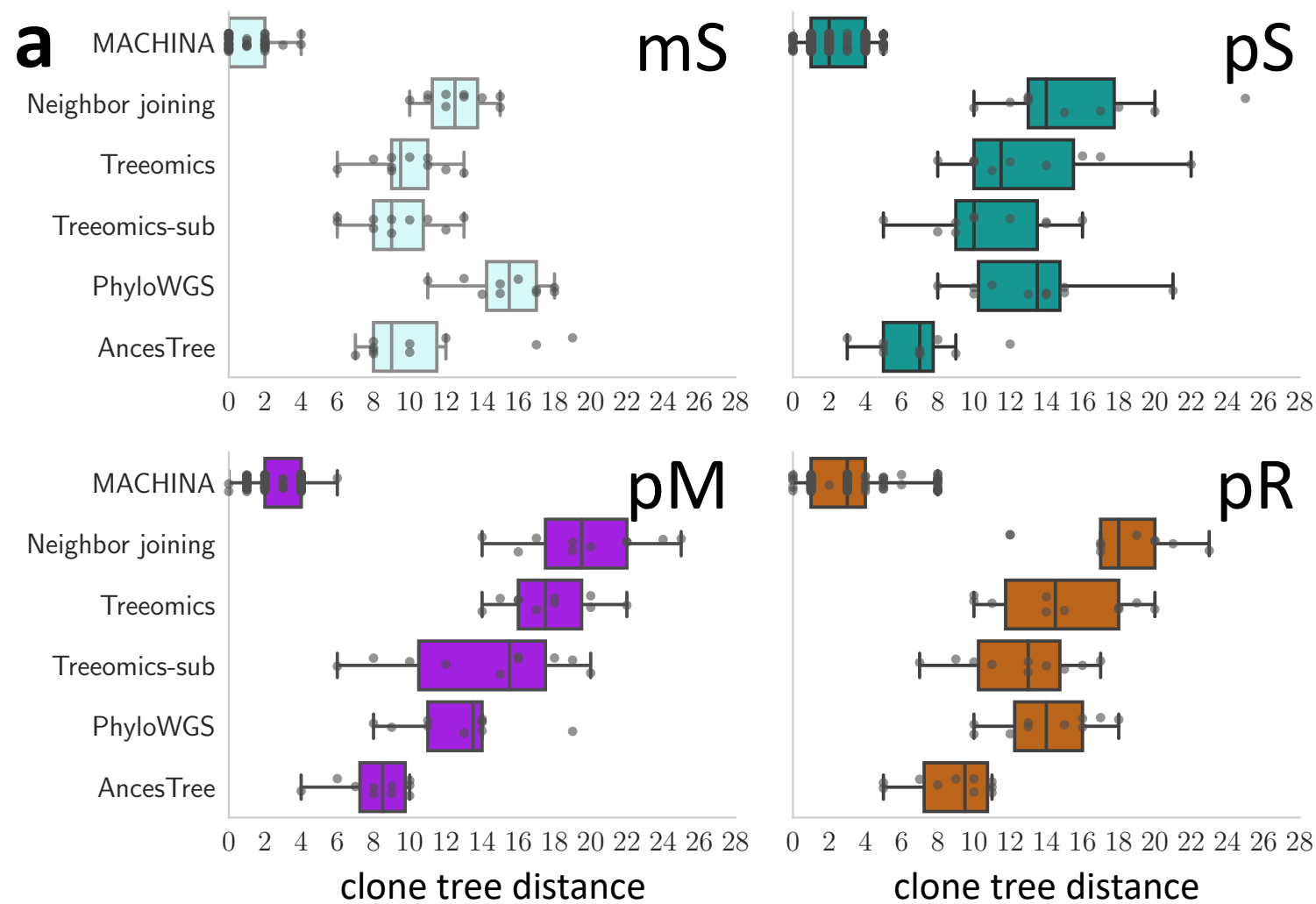
Label ancestral vertices by anatomical sites

Resolve clone tree ambiguities

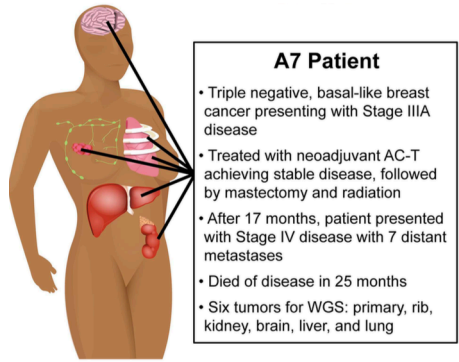
### Migration Graph



# MACHINA accurately infers clone trees and migration histories on simulated data



# Applying MACHINA to Metastatic Breast Cancer



## A7 Patient

- Triple negative, basal-like breast cancer presenting with Stage IIIA disease
- Treated with neoadjuvant AC-T achieving stable disease, followed by mastectomy and radiation
- After 17 months, patient presented with Stage IV disease with 7 distant metastases
- Died of disease in 25 months
- Six tumors for WGS: primary, rib, kidney, brain, liver, and lung

Hoadley et al.  
Tumor Evolution in Two Patients with Basal-like Breast Cancer: A Retrospective Genomics Study of Multiple Metastases.  
*PLOS Med*, 13(12) 2016

