

## Lecture 7

Sunday, September 23, 2018 4:02 PM

### Protein Structure Alignment

Let  $\Sigma$  be the alphabet of amino acids, i.e.  $|\Sigma| = 20$ .

Let  $A \in \Sigma^m$  and  $B \in \Sigma^n$ .

We are given  $d_A : [m] \times [m] \rightarrow \mathbb{R}^{\geq 0}$   
 $d_B : [n] \times [n] \rightarrow \mathbb{R}^{\geq 0}$ .

We are given a distance threshold  $\tau \geq 0$ .

We obtain  $C^A = [c_{ij}^A]$  where

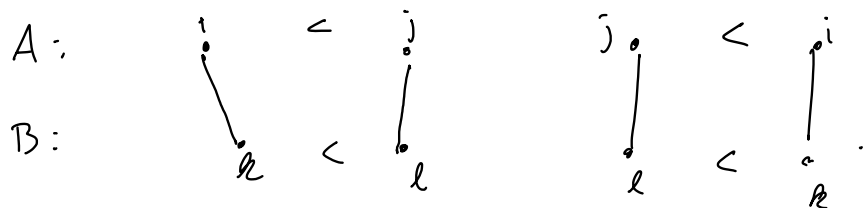
$$c_{ij}^A = 1 \iff d_A(i, j) \leq \tau.$$

$C^B = [c_{\ell, \ell'}^B]$  is symmetric.

An alignment  $S$  of  $A$  and  $B$  is a

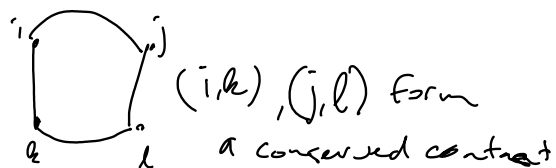
set of non-overlapping pairs  $(i, \ell), (j, \ell') \in S$  with the constraint that

$$(i < j \text{ and } \ell < \ell') \text{ or } (j < i \text{ and } \ell' < \ell)$$



A conserved contact in  $S$  is defined by two aligned pairs  $(i, \ell), (j, \ell') \in S$  s.t.

$$c_{i,j}^A = 1 \quad \text{and} \quad c_{k,l}^B = 1.$$



Problem. [Contact Map Overlap (CMO)]

Given contact maps  $C^A$  and  $C^B$ , find an alignment of  $A$  and  $B$  with maximum number of conserved contacts.

We can view CMO as a graph problem.

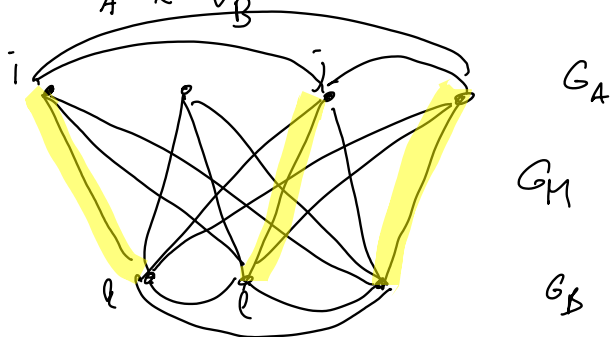
There are two graph representations.

$$\begin{cases} G_A = (V_A, E_A) & |V_A| = m \\ (v_i, v_j) \in E_A \Leftrightarrow c_{i,j}^A = 1 \end{cases}$$

$G_B = (V_B, E_B)$  is symmetric

① Matching graph  $G_M = (V_A \cup V_B, E_M)$

$$E_M = V_A \times V_B$$



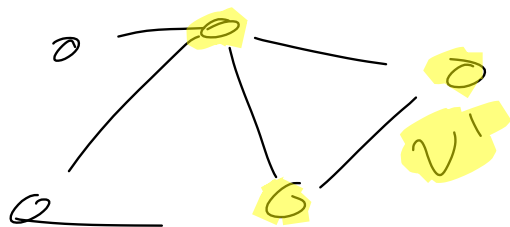
An alignment  $S$  is a non-crossing matching in  $G_M$ , i.e. for all distinct edges  $(v_i, v_k), (v_j, v_l) \in S$  it holds that

$$(i < j \text{ and } k < l) \text{ or } (j < i \text{ and } l < k).$$

Given a simple graph  $G=(V,E)$

Clique is a subset  $V' \subseteq V$  of vertices s.t.

For all distinct  $u,v \in V'$  it holds that  $(u,v) \in E$



$$|V|=n \quad |E| \leq \binom{n}{2} = O(n^2)$$

### CLIQUE problem

Given  $G=(V,E)$  and  $k \in \mathbb{N}$ ,

Is there a clique in  $G$  of size  $k$ ?

NP-complete

There is no algorithm  
with running time  $O(p_d(n))$

$\rightarrow n^c$  where  $c > 0$

unless  $P=NP$ .